143. *On the Cauchy Problem for the Equation with Multiple Characteristic Roots*

By Tadayoshi KANO

Faculty of Science, Osaka University

(Comm. by Kinjirô KUNUGI, M.J.A., Sept. 12, 1967)

1. Introduction. 1.1. S. Mizohata [1] obtained the necessary condition for the well posedness in Petrowsky's sense of the Cauchy probem for

$$M[u]=\frac{\partial}{\partial t}u-\sum_{j=1}^{n}A_j(x,t)\frac{\partial}{\partial x_j}u$$

where $\{A_j(x,t)\}$ are $N\times N$ matrices which are bounded and sufficiently smooth in x and t.

In [1] the first approximation to M plays an important part. M is approximated by the singular integral operator associated with tangential operator.

Now we consider the higher order approximation to differential operator in some sense, and get a result presented in the following paragraphs.

1.2. Consider the differential operator

$$L=\left(\frac{\partial}{\partial t}\right)^m+\sum_{\substack{|\nu|+j\leqq m\\ j\leqq m-1}}a_{\nu,j}(x,t)\left(\frac{\partial}{\partial x}\right)^{\nu}\left(\frac{\partial}{\partial t}\right)^{j} \tag{1}$$

where

$$x=(x_1,\cdots,x_n),\qquad \left(\frac{\partial}{\partial x}\right)^{\nu}=\left(\frac{\partial}{\partial x_1}\right)^{\nu_1}\cdots\left(\frac{\partial}{\partial x_n}\right)^{\nu_n}$$

and $\{a_{\nu,j}(x,t)\}$ are contained in $\mathscr{B}_{x,t}$.

We denote the principal part of L by

$$L_0=\left(\frac{\partial}{\partial t}\right)^m+\sum_{\substack{|\nu|+j=m\\ j\leqq m-1}}a_{\nu,j}(x,t)\left(\frac{\partial}{\partial x}\right)^{\nu}\left(\frac{\partial}{\partial t}\right)^{j} \tag{2}$$

and associate the characteristic equation to it:

$$L_0(x,t,\xi;\lambda)=\lambda^m+\sum_{\substack{|\nu|+j=m\\ j\leqq m-1}}a_{\nu,j}(x,t)\xi^{\nu}\lambda^{j}=0 \tag{3}$$

where $\xi^{\nu}=\xi_1^{\nu_1}\cdots\xi_n^{\nu_n}$.

1.3. We consider the Cauchy problem for (1) in L^2 sense.

Definition. *The Cauchy problem for* (1) *is said to be well posed in* L^2 *sense if there exists a unique solution* $u=u(x,t)$ *of* $Lu=0$ *such that*

$$u(x,t)\in\mathcal{E}_t^0(\mathcal{D}_{L^2}^{m-1})\cap\cdots\cap\mathcal{E}_t^{m-1}(L^2),\ (0\leqq t\leqq T) \tag{4}$$

for any initial data Ψ