142. *Eigenfunction Expansions Associated with the Schrödinger Operator with a Complex Potential and the Scattering Inverse Problem*

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1. Introduction. In this note[1] we are concerned with the Schrödinger operator $-\Delta+q(x)$ acting in the Hilbert space $\mathfrak{H}=L^2(E_3)$, where E_3 denotes the 3-dimensional Euclidean space. We consider the case where $q(x)$ is a complex-valued potential function assumed to satisfy the following conditions:

(A) $q(x)\in L^2(E_3)$, is locally Hölder continuous except for a finite number of singularities and behaves like $O(|x|^{-2-\delta})(\delta>0)$ as $|x|\to\infty$.

The eigenfunction expansion theorem associated with $-\Delta+q(x)$ was already proved, based on a work of Povzner [7], by Ikebe [1] under the same assumptions on $q(x)$ when it is real-valued. Our purpose is to extend his results to the case of complex-valued potentials. We use the methods developed by J. Schwartz [8], Kato [3], and Kuroda [4], [5], and follow almost the same line of the proof given by Ikebe. In our case, however, the existence of a uniformly bounded spectral resolution $E(e)$ of $-\Delta+q(x)$ is not proved if we choose real intervals e arbitrarily. So our results on the expansion problem will become rather of a local character.

The expansion formula can be applied to solve the scattering inverse problem formulated by Faddeev in [2]. His result is the following: A real-valued potential function $q(x)$ can be determined uniquely, under the assumptions that $q(x)\in C^1(E_3)$ and

(A_1) $$q(x)=O(|x|^{-3-\delta})(\delta>0) \text{ as } |x|\to\infty,$$

from the assymptotic conditions for $|k|\to\infty$ of the function $\theta_\pm(n,\nu;|k|)$ having a physical meaning.[2] We shall extend this result also to the case of complex-valued potential assumed to satisfy (A_1) in addition to (A). In our proof it is not necessary to assume $q(x)\in C^1(E_3)$.

2. Spectral resolutions. We consider $-\Delta+q(x)$ to be defined on $C_0^\infty(E_3)$. We denote by L_0 the selfadjoint extension of $-\Delta$ with

1) The detailed proof of the following results will be given in a forthcoming paper.

2) $|\theta_-|^2$ gives the so-called differential closs section of scattering for the particle incident in the direction ν and scattered in the direction n. For the definition of $\theta_\pm(n,\nu;|k|)$ see (25) in § 4.