137. *A Theorem on Paracompactness of Product Spaces*

By Yûkiti KATUTA

Ehime University

(Comm. by Kinjirô KUNUGI, M.J.A., Sept. 12, 1967)

1. Introduction. As is well known, the product space of two paracompact Hausdorff spaces is not normal in general. In [4], K. Morita has proved the following:

Let X be a paracompact normal space which is a countable union of locally compact closed subsets, and let Y be a paracompact normal space. Then the product space $X\times Y$ is paracompact and normal.

The purpose of this note is to prove a theorem which is a generalization of Morita's result mentioned above.

Definition. A collection $\{A_\lambda \mid \lambda\in\Lambda\}$ of subsets of a topological space is called *order locally finite*, if we can introduce a total order $<$ in the index set Λ such that for each $\lambda\in\Lambda$ $\{A_\mu \mid \mu<\lambda\}$ is locally finite at each point of A_λ.

Theorem. *If a regular space X has two coverings $\{C_\lambda \mid \lambda\in\Lambda\}$ and $\{U_\lambda \mid \lambda\in\Lambda\}$ such that*

i) *C_λ is compact, U_λ is open and $C_\lambda\subset U_\lambda$ for each $\lambda\in\Lambda$, and*

ii) *$\{U_\lambda \mid \lambda\in\Lambda\}$ is order locally finite,*

then for any paracompact regular space Y the product space $X\times Y$ is paracompact.

Let X be a paracompact regular space which is a countable union of locally compact closed subsets. Then there exists a σ-locally finite covering $\{C_\lambda \mid \lambda\in\Lambda\}$ of X such that each C_λ is compact. Moreover, since X is paracompact, there exists a σ-locally finite open covering $\{U_\lambda \mid \lambda\in\Lambda\}$ of X such that U_λ contains C_λ for each $\lambda\in\Lambda$ (see [4]). By Lemma 1 below, a σ-locally finite collection is order locally finite. Therefore our theorem covers certainly Morita's result (in the case when X and Y are regular spaces).

Recently T. Ishii [2] has proved the following:

Let X be the image under a closed continuous mapping of a locally compact and paracompact Hausdorff space, and let Y be a paracompact Hausdorff space. Then the product space $X\times Y$ is a paracompact Hausdorff space.

He has also showed that his result is not covered by Morita's result. As the example below shows, our theorem is not contained