134. On Some Classes of Operators. I

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(Comm. by Kinjirô KUNUGI, M.J.A., Sept. 12, 1967)

Introduction. Generalizing the concept of normality, several authors have introduced classes of non-normal operators.

Thus, one of these classes is the class of hyponormal operators of P. Halmos [1]. In the paper [2] it was introduced a new class of operators generalizing hyponormal operators.

It is the aim of this Note to introduce a new class of operators which generalizes the class of operators of class (N) of [2] and give some properties. The definition and some properties has sense also for operators on Banach spaces, however we consider only Hilbert spaces operators.

1. Let T be a bounded linear operator on a Hilbert space H.

The operator is of class (N) if

$$x \in H, \qquad \|x\|=1, \qquad \|Tx\|^2 \leq \|T^2x\|.$$

This definition suggests the following

Definition 1. The operator T is of class (N) and order k if

$$x \in H, \qquad \|x\|=1 \qquad \|Tx\|^k \leq \|T^kx\|.$$

We write this as $T \in \mathcal{C}(N, k)$. It is clear that the operators of class (N) is $\mathcal{C}(N, 2)$.

Theorem 1. *If $T \in \mathcal{C}(N, k)$, then the spectral radius of T, γ_T is equal to $\|T\|$.*

Proof. By definition there exists a sequence $\{x_n\}$, $\|x_n\|=1$ such that

$$\|Tx_n\| \rightarrow \|T\|=1.$$

(We may suppose, without loss of generality that $\|T\|=1$.) Since, for every x, $\|x\|=1$,

$$\|Tx\|^k \leq \|T^kx\|$$

we have

$$\lim \|T^kx_n\|=1$$

This leads, also, to

$$\lim \|T^jx_n\|=1 \qquad 1 \leqslant J \leqslant K.$$

Since

$$\|T^{k+1}x\| = \left\| T^k \frac{Tx}{\|Tx\|} \right\| \|Tx\| \geqslant \|T^2x\|^k \frac{1}{\|Tx\|^{k-1}}$$

If we put in this inequality, $x=x_n$ we obtain

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