129. *Locally Symmetric K-Contact Riemannian Manifolds*

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§ 1. **Introduction.** Let M be a contact Riemannian manifold with a contact form η, the associated vector field ξ, a (1, 1)-tensor ϕ and the associated Riemannian metric g:

$$\phi^i_j\xi^j=0, \qquad \eta_i\phi^i_j=0, \qquad \eta_i\xi^i=1,$$
$$\phi^i_j\phi^j_k=-\delta^i_k+\xi^i\eta_k,$$
$$g_{ij}\xi^j=\eta_i,$$
$$g_{ij}\phi^i_r\phi^j_s=g_{rs}-\eta_r\eta_s.$$

If ξ is a Killing vector field with respect to g, M is called a K-contact Riemannian manifold, and then ξ is an infinitesimal automorphism of this structure. And we have

(1.1) $$\nabla_j\xi^i=-\phi^i_j,$$

(1.2) $$R_{jk}\xi^k=(m-1)\eta_j, \qquad m=\dim M,$$

(1.3) $$\eta_r R^r{}_{jks}\xi^s=g_{jk}-\eta_j\eta_k.$$

Further if the following relation

(1.4) $$\eta_r R^r{}_{jkl}=g_{jk}\eta_l-g_{jl}\eta_k$$

is satisfied, M is called a Sasakian manifold ([2]).

§ 2. **Statement of results.** M is said locally symmetric if we have $\nabla_h R^i{}_{jkl}=0$. In this paper first in § 3 we prove the following

Theorem 1. *Any locally symmetric K-contact Riemannian manifold is Sasakian and has constant curvature* 1.

As an immediate consequence we have

Theorem 2. *Any complete, simply connected and locally symmetric K-contact Riemannian manifold is globally isometric with a unit sphere.*

Any compact semi-simple Lie group G has the positive definite Riemannian metric defined by the Killing form, which is locally symmetric and is not of constant curvature, provided dim $G>3$ (cf. [4], p. 122). Therefore we get

Corollary 3. *A compact semi-simple Lie group G* (dim $G>3$) *with the usual metric can not be a K-contact Riemannian manifold.*

Remark 4. With regard to this Corollary in [1] (p. 729) it was shown that, if G is a semi-simple Lie group on which a left G-invariant contact form η is defined, then G is 3-dimensional and locally isomorphic with either $O(3)$ or $SL(2)$.