166. On Closed Mappings and M-Spaces. I

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1. Introduction. Recently K. Morita [1] has introduced the notion of M-spaces. We shall say that a topological space X is an M-space if there exists a normal sequence $\{\mathfrak{U}_n \mid n=1, 2, \cdots\}$ of open coverings of X which satisfies the condition below:

(*) If a family $\mathfrak{K}$ consisting of a countable number of subsets of X has the finite intersection property and contains as a member a subset of $\mathrm{St}(x_0, \mathfrak{U}_n)$ for every n and for some fixed point x_0 of X, then $\bigcap\{\bar{K} \in \mathfrak{K}\} \neq \phi$.

In this paper we shall introduce the notion of M^*-spaces which contains all M-spaces, and study some properties of these spaces. We shall say that a topological space X is an M^*-space if there exists a sequence $\{\mathfrak{F}_n \mid n=1, 2, \cdots\}$ of locally finite closed coverings of X which satisfies the condition (*). Of course we can assume without loss of generality that $\mathfrak{F}_{n+1}$ is a refinement of $\mathfrak{F}_n$ for every n. Theorems 2.3 and 2.4 will play the important roles in the proof of the main theorem which will be mentioned in the following paper "On closed mappings and M-spaces. II".

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2. Some properties of M*-spaces. **Lemma 2.1.** *Let f be a closed continuous mapping of a T_1-space X onto a topological space Y. If $f^{-1}(y)$ is countably compact for any point y of Y, and if $\{F_\lambda \mid \lambda \in \Lambda\}$ is a locally finite collection of closed subsets of X, then $\{f(F_\lambda) \mid \lambda \in \Lambda\}$ is also a locally finite collection of closed subsets of Y.*

This lemma is due to A. Okuyama [4].

Lemma 2.2. *Let X be an M^*-space with a sequence $\{\mathfrak{F}_n\}$ of locally finite closed coverings of X such that $\{\mathfrak{F}_n\}$ satisfies the condition (*) and that $\mathfrak{F}_{n+1}$ is a refinement of $\mathfrak{F}_n$ for every n, and C any countably compact subset of X, where X is T_1. If $\mathfrak{K}$ is a family of countable number of subsets of X which has the finite intersection property and contains as a member a subset of $St(C, \mathfrak{F}_n)$ for every n, then $\bigcap\{\bar{K} \mid K \in \mathfrak{K}\} \neq \phi$.*

Proof. First we note that, if $\mathfrak{F}$ is any locally finite closed covering of X, then a countably compact subset C of X intersects with only finite members of $\mathfrak{F}$. Hence for every n, C intersects