195. *On Free Abelian m-Groups.* III

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In this part, the notion of tensor product of abelian m-groups will be introduced.

Definition. The *tensor product* of the abelian m-groups M and N is defined as F/θ and is denoted by $M\boxtimes N$.

If $|(x, y)|/\theta$ is denoted by $x\boxtimes y$, observe that

$$[x_1x_2\cdots x_m]\boxtimes y=[(x_1\boxtimes y)(x_2\boxtimes y)\cdots(x_m\boxtimes y)],$$
$$x\boxtimes[y_1y_2\cdots y_m]=[(x\boxtimes y_1)(x\boxtimes y_2)\cdots(x\boxtimes y_m)],$$

and $x^{\langle n\rangle}\boxtimes y=x\boxtimes y^{\langle n\rangle}=(x\boxtimes y)^{\langle n\rangle}$.

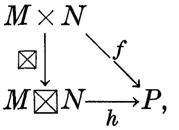
Theorem 9. *Let M, N, P be arbitrary abelian m-groups and $f\colon M\times N\to P$ be a function satisfying the conditions*

(a) $f([x_1x_2\cdots x_m], y)=[f(x_1, y)f(x_2, y)\cdots f(x_m, y)]$,

(b) $f(x, [y_1y_2\cdots y_m])=[f(x, y_1)f(x, y_2)\cdots f(x, y_m)]$,

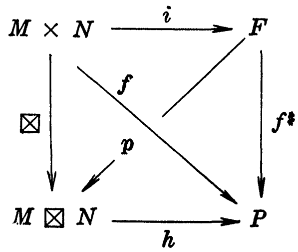
(c) $f(x^{\langle n\rangle}, y)=f(x, y^{\langle n\rangle})$,

for all $x, x_1, \cdots, x_m\in M$ and $y, y_1, \cdots, y_m\in N$. Then there exists uniquely an m-group homomorphism $h\colon M\boxtimes N\to P$ such that the following diagram is commutative



that is, $h(x\boxtimes y)=f(x, y)$ for all $x\in M$ and $y\in N$.

Proof. Let F be the free abelian m-group on $M\times N$ and $i\colon M\times N\to F$ be the injection $i(x, y)=|(x, y)|$. Consider the following diagram.



By Theorem 4, f possesses a unique homomorphic extension $f^{\#}\colon F\to P$ such that $f^{\#}\cdot i(x, y)=f(x, y)$ so that $f^{\#}(|(x, y)|)=f(x, y)$. Since

$$f^{\#}(|([x_1x_2\cdots x_m], y)|)=f([x_1x_2\cdots x_m], y)$$
$$=[f(x_1, y)f(x_2, y)\cdots f(x_m, y)]=[f^{\#}(|(x_1, y)|)\cdots f^{\#}(|(x_m, y)|)],$$