192. *A Note on M-Spaces*

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Let X and Y be topological spaces and f be a closed, continuous mapping from X onto Y such that for each $y \in Y$, $f^{-1}(y)$ is countably compact. Such an f is called a quasi-perfect mapping. Moreover, if $f^{-1}(y)$ is compact, f is called a perfect mapping. In [5], K. Morita defined P-spaces and M-spaces. According to [5] an M-space is a P-space, and in order that X be an M-space it is necessary and sufficient that there exist a metric space Y and a quasi-perfect mapping from X onto Y. We study the M-spaces with some compactness or completeness by use of the above mappings. Let X be a completely regular T_1 space which admits a complete uniformity. Then X is not necessarily paracompact by H. H. Corson [2]. In this note, it will be shown that X is paracompact if the space X concerned is an M-space. The author wishes to thank Prof. K. Nagami who has given useful advices. In this paper, for topological spaces no separation axiom is assumed unless otherwise provided.

Theorem 1. *Every M-space is countably paracompact.*

Theorem 2. *If X is a pseudo-compact M-space, then X is countably compact. Therefore in a pseudo-compact space, to be an M-space is equivalent to countable compactness.*

As will be seen later, a P-space is not countably paracompact in general and a pseudo-compact P-space need not be countably compact.

Proof of Theorem 1. The result follows from the above characteristic property of M-spaces and the following lemma.

Lemma. *If f is a quasi-perfect mapping from X onto Y and Y is countably paracompact* (*or countably compact*), *then X is countably paracompact* (*or countably compact, respectively*).

This lemma is known in [4].

Proof of Theorem 2. Let f be a quasi-perfect mapping from X onto a metric space Y. Since Y is pseudo-compact metric, Y is compact. Hence X is countably compact by the above lemma.

It is known in [5] every countably compact space is an M-space and every normal $P(1)$-space is countably paracompact. It is to be noted that *a pseudo-compact, locally compact Hausdorff P-space is not necessarily countably paracompact and hence not an M-space.* The Tychonoff plank $X=[0, \omega_1]\times[0, \omega]-t$, where t is the point