190. *Partially Ordered Sets and Semi-Simplicial Complexes*

By Sigeru OKAMOTO

Department of Mathematics, Yamaguchi University

(Comm. by Kinjirô KUNUGI, M.J.A., Nov. 13, 1967)

§1. **Introduction.** Let $\mathcal{M}$ be the category of partially ordered sets and isotone maps, and $\mathcal{S}$ the one of s.s. (semi-simplicial) complexes and s.s. maps.

Then, a covariant functor L: $\mathcal{M}\to\mathcal{S}$ is defined naturally as follows:

For a partially ordered set X, let $M(X)$ be the ordered simplicial complex whose n-simplex is an ordered sequence $(x_0, x_1, \cdots, x_n)$ for $x_i \in X$ and $x_0 < x_1 < \cdots < x_n$, and define $L(X)$ as the ordered s.s. complex of $M(X)$.

The object of this note is to discuss on the fundamental properties of L. It is shown that two partially ordered sets X and Y are isomorphic if and only if $L(X)$ and $L(Y)$ are s.s. isomorphic (Corollary 6). Also, we can define the notion of "homotopy" so that X and Y are homotopy equivalent if and only if $L(X)$ and $L(Y)$ are so (Corollary 8).

Furthermore, a (co)homology group of a pair (X, A) of a partially ordered set and its ideal can be defined by the one of the s.s. pair $(L(X), L(A))$, and the seven axioms of Eilenberg-Steenrod ([2]) are satisfied (Theorem 10).

It is interesting that $L(X)$ satisfies the extension condition for the dimension >1(Theorem 4). Here, we notice that there does not necessarily exist a partially ordered set X such that $M(X)$ is simplicially isomorphic to a given simplicial complex K.

Full details will be appear elsewhere.

§2. **Fundamental properties of L.** For the terminology and the notations concerning the partially ordered sets or the s.s. (semi-simplicial) complexes, see [1] or [5] respectively.

For a partially ordered set X, and s.s. complex $L(X)$ is defined as follows:

An n-simplex of $L(X)$ is an ordered sequence $(x_0, \cdots, x_n)$ where $x_i \in X$ and $x_0 \leqq \cdots \leqq x_n$. The face- and degeneracy-operators are given by

$$\partial_i(x_0, \cdots, x_n) = (x_0, \cdots, x_{i-1}, x_{i+1}, \cdots, x_n),$$
$$s_i(x_0, \cdots, x_n) = (x_0, \cdots, x_i, x_i, \cdots, x_n),$$