188. Representation Ring of Lie Group F_4

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1. **Introduction.** The aim of this paper is to determine the representation ring $R(F_4)$ of group F_4, which is a simply connected compact simple Lie group of exceptional type F. Let $\mathfrak{J}$ denote the Jordan algebra consisting of all 3-hermitian matrices over the division ring of Cayley numbers. The group F_4 is obtained as the automorphism group of $\mathfrak{J}$. Let $\mathfrak{J}_0$ be the set of all elements of $\mathfrak{J}$ with zero trace. Then $\mathfrak{J}_0$ is invariant by the operation of F_4. Thus we have an F_4-C-module $\mathfrak{J}_0\otimes_R C$.[1] On the other hand, we know another F_4-C-module $\mathfrak{F}_4\otimes_R C$, where $\mathfrak{F}_4$ is the Lie algebra of F_4. The result is as follows: *$R(F_4)$ is a polynomial ring $Z[\lambda_1, \lambda_2, \lambda_3, \mu]$ with 4 variables $\lambda_1, \lambda_2, \lambda_3$, and μ, where λ_i is the class of the exterior F_4-C-module $\Lambda^i(\mathfrak{J}_0\otimes_R C)$ in $R(F_4)$ for $i=1, 2, 3$, and μ is the class of $\mathfrak{F}_4\otimes_R C$ in $R(F_4)$.* In this paper, we shall describe the outline of our methods; these may be analogous to those as in the cases of classical groups [1] and of group G_2 [2]. The details will appear in the Journal of the Faculty of Science, Shinshu University, vol. 3, 1968.

2. **Representation ring.** Let G be a topological group. Let $M(G)$ denote the set of all G-C-isomorphism classes of G-C-modules. The direct sum $V\oplus W$ and the tensor product $V\otimes W$ of two G-C-modules V, W define a semiring structure on $M(G)$. The representation ring $R(G)=(R(G), \phi)$ (where ϕ: $M(G)\to R(G)$ is a semiring homomorphism) is the universal ring associated with the semiring $M(G)$.

3. **Jordan algebra $\mathfrak{J}$, group F_4 and Lie algebra $\mathfrak{F}_4\otimes_R C$.**

Let $\mathfrak{C}$ denote the division ring of Cayley numbers and $\mathfrak{J}$ be the set of all 3-hermitian matrices X over $\mathfrak{C}$. In $\mathfrak{J}$, we define a Jordan multiplication by

$$X\circ Y=\frac{1}{2}(XY+YX).$$

Then $\mathfrak{J}$ is a 27-dimensional commutative distributive (non-associative) algebra over R. Let F_4 denote the group of all automorphisms of $\mathfrak{J}$. As is well known, F_4 is a simply connected compact simple Lie group of exceptional type F. Obviously, $\mathfrak{J}$ is an F_4-R-module.

1) R and C are the fields of real and complex numbers, respectively.