187. A Note on Almost-Countably Paracompact Spaces

By M. K. SINGAL and Asha RANI
University of Delhi, India

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In a recent paper [1] the concept of almost-countably paracompact spaces has been introduced. A space X is said to be almost-countably paracompact if for each countable open covering $\mathcal{A}$ of X there exists a locally-finite family $\mathcal{B}$ of open subsets of X which refines $\mathcal{A}$ and the family of closures of members of $\mathcal{B}$ forms a covering of X. In the present note we give a characterization of such spaces.

A° denotes the interior of A and $\bar{A}$ denotes the closure of A.

Theorem 1. *For a topological space (X, ζ) the following are equivalent*:

(a) *X is almost-countably paracompact.*

(b) *For every decreasing sequence $\{F_i\}$ of closed subsets of X such that $F_i^0 \neq \phi$ for all i and $\bigcap_{i\in N} F_i \subset U$ where U is open, there exists a decreasing sequence $\{G_i\}$ of open subsets of X such that $F_i^0 \subset \bar{G}_i$ for each i and $\bigcap_{i\in N} \bar{G}_i \subset \bar{U}$.*

(c) *For each decreasing sequence $\{F_i\}$ of closed subsets of X such that $F_i^0 \neq \phi$ for each i and $\bigcap_{i\in N} F_i \subset U$ where U is open, there exists a decreasing sequence $\{H_i\}$ of closed subsets of X such that $F_i^0 \subset H_i$ for each i and $\bigcap_{i\in N} H_i \subset \bar{U}$.*

Proof. $(a)\Rightarrow(b)$. Since $\bigcap_{i\in N} F_i \subset U$, therefore $X\sim U \subset X \sim \bigcap_{i\in N} F_i = \bigcup_{i\in N} X\sim F_i$. Thus $\{X\sim F_i : i \in N\} \cup \{U\}$ is a countable open covering of X. Therefore there exists a locally-finite family $\{V_i\}$ of open subsets of X such that $V_i \subset X\sim F_i$ for each i and $(\bigcup_{i\in N} \bar{V}_i) \cup \bar{U} = X$. For each i let $G_i = \bigcup_{n=i+1}^{\infty} (V_n \cup U)$. Then $\bar{G}_i \supset X \sim (\bar{V}_1 \cup \cdots \cup \bar{V}_i) \supset X \sim \overline{X\sim F_i} = F_i^0$. Thus $\{G_i\}$ is a decreasing sequence of open sets such that $F_i^0 \subset \bar{G}_i$. We shall show now that $\bigcap_{i\in N} \bar{G}_i \subset \bar{U}$. If a point $x \in \bigcap_{i\in N} \bar{G}_i$ and $x \notin \bar{U}$, then $\{\bar{V}_i\}$ being locally-finite there exists an open set M_x which intersects finitely many sets $\bar{V}_i^s$. Therefore there exists an integer i such that $M_x \cap \left(\bigcup_{n=i+1}^{\infty} \bar{V}_n\right) = \phi$. Therefore $x \notin \overline{\bigcup_{n=i+1}^{\infty} \bar{V}_n}$. Also, $x \notin \bar{U}$. Therefore $x \notin \bar{G}_i$, which is a contradiction. Therefore $x \in \bigcap_{i\in N} \bar{G}_i$, which implies $x \in \bar{U}$. Thus $\bigcap_{i\in N} \bar{G}_i \subset \bar{U}$.

$(b)\Rightarrow(c)$. This is obvious since we can take $H_i = \bar{G}_i$.

$(c)\Rightarrow(a)$. Let $\{U_i\}$ be any countable open covering of X. For