186. On the Representations of SL(3, C). I

By Masao TSUCHIKAWA

(Comm. by Kinjirô KUNUGI, M.J.A., Nov. 13, 1967)

1. We shall determine the intertwinning operators and the equivalence relation among the representations of the group $SL(3, C)$, generalizing the method described in [1] for $SL(2, C)$. We denote by G the group $SL(3, C)$ and we adopt the notations of the book [2] thoughout this paper, but elements of Z will be denoted by

$$z=\begin{bmatrix}1 & & \\ z_1 & 1 & \\ z_3 & z_2 & 1\end{bmatrix}, \quad \text{especially} \quad z_1=\begin{bmatrix}1 & & \\ z_1 & 1 & \\ & & 1\end{bmatrix}$$

and so on. Let W be the Weyl group of G consisted of $s_0=e$, s_1, s_2, $s_3=s_2s_1$, $s_4=s_1s_2$ and $s_5=s_1s_2s_1=s_2s_1s_2$, where

$$s_1=\begin{bmatrix} & 1 & \\ 1 & & \\ & & -1\end{bmatrix}, \qquad s_2=\begin{bmatrix}-1 & & \\ & & 1 \\ & 1 & \end{bmatrix}.$$

Let G^0 be the set of all g such that $g_{33}\cdot g^{11}\neq 0$, then $g=kz$ for all $g\in G^0$.

2. Let χ be an integral character of D: $\chi(\delta)=(\delta_2\delta_3)^{(l_1,m_1)}\delta_3^{(l_2,m_2)}$ ($l_k, m_k>0$), and $\mathcal{E}_\chi$ be the finite dimensional vector space of polynomials φ on Z which are at most of degree (l_1-1, m_1-1) with respect to $z_1, z_1z_2-z_3$ and of degree (l_2-1, m_2-1) with respect to z_2, z_3. Then, according to the theorem of Cartan and Weyl, for every finite dimensional irreducible representation of G there exists χ such that given representation E^χ is realized on $\mathcal{E}_\chi$ by $E_g^\chi\varphi(z)=\chi\beta^{-1/2}(k_g)\varphi(z_g)$.

Now let $\chi=(\lambda, \mu)$ be a complex character of D: $\chi(\delta)=(\delta_2\delta_3)^{(\lambda_1,\mu_1)}\delta_3^{(\lambda_2,\mu_2)}$ (λ_k, μ_k are complex numbers and $\lambda_k-\mu_k$ are integers), then we can construct a representation $\{T^\chi, \mathcal{D}_\chi\}$ as follows. Let $\mathcal{D}_\chi$ be the vector space of C^∞-functions φ on Z, satisfying the condition that for every $s\in W$ $\varphi_s(z)=\chi\beta^{-1/2}(k_s)\varphi(z_s)$ is also a C^∞-function. The topology of $\mathcal{D}_\chi$ is defined by the compact uniform convergence of every derivative for every φ_s ($s\in W$). The operator T_g^χ on $\mathcal{D}_\chi$ is defined by $T_g^\chi\varphi(z)=\chi\beta^{-1/2}(k_g)\varphi(z_g)$. This representation is identical with the induced representation $T^\chi=Ind\{\chi\,|\,K\rightarrow G\}$. If all λ_k, μ_k are positive integers, the representation $\{E^\chi, \mathcal{E}_\chi\}$ is contained in $\{T^\chi, \mathcal{D}_\chi\}$ as a sub-representation.

3. Let $B(\varphi, \psi)$ be a continuous bilinear form on $\mathcal{D}_\chi\times\mathcal{D}_{\chi'}$ such