185. *A Note on the Generation of Nonlinear Semigroups in a Locally Convex Space*

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1. Let X be an (FM)-space, i.e., a Frechet space which is also a Montel space. For example, the space $H(\Omega)$ of holomorphic functions on a domain Ω in the complex plane, which is endowed with the topology of locally uniform convergence, the space $(\mathfrak{S})$ of rapidly decreasing functions on R^n and R^n are this space.

For a not necessarily linear operator A from X into itself, we introduce the following conditions:

(1) There exists a positive constant $\delta > 0$ such that for each $h \in (0, \delta]$, the topological inverse mapping $(I-hA)^{-1}$ of the mapping $x \to x - hAx$ exists on X as a single valued operator.

(2) For any $T > 0$, the family of operators $\{(I-hA)^{-n}\}$ is equicontinuous on X in $h \in (0, \delta]$ and n with $hn \in [0, T]$. (Put $(I-hA)^0 = I$, the identity mapping.)

(3) For any $x \in D(A)$ and for any $T > 0$, the set $\{A(I-hA)^{-n}x: h \in (0, \delta], hn \in [0, T]\}$ is bounded in X.

Definition 1. *A not necessarily linear operator A from X to itself is said to be of class $\mathfrak{A}$ if for this A all of the above conditions are satisfied.*

In the case that A is a densely defined closed linear operator, the well-known necessary and sufficient condition for A being the infinitesimal generator of an equicontinuous semigroup is rather stronger than the condition $A \in \mathfrak{A}$. We mention here some remarks on the abovementioned conditions:

(i) From (3) it follows that for any $x \in X$ the set $\{(I-hA)^{-n}x: h \in (0, \delta], hn \in [0, T]\}$ is bounded in X.

(ii) From (2) it follows that if $D(A) \ni x_n \to x$ and $Ax_n \to y$, then $x \in D(A)$ and $Ax = y$.

(iii) The following condition implies (2) and (3):

For any $x \in X$ and $T > 0$ there exists a neighbourhood $U(x)$ of x such that for any continuous seminorm p there exists a continuous seminorm q which is independent of $h \in (0, \delta]$, n with $hn \in [0, T]$ and $z \in U(x)$, such that

$$p((I-hA)^{-n}x - (I-hA)^{-n}z) \leqq q(x-z), \qquad z \in U(x).$$

(iv) If A maps bounded sets in $D(A)$ into bounded sets, then