183. *An Important Relation in Homotopy Groups of Spheres*

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1. In computing the p-primary component ${}_pG_k$ of the k-stem group $G_k=\lim \pi_{n+k}(S^n)$, p denoting always an odd prime, an essential difficulty lies in the case $k=2p^2(p-1)-3$. Recently, Cohen [1] has announced that ${}_pG_{2p^2(p-1)-3}\approx Z_p$, which is equivalent to say that $\alpha_1\beta_1^p\neq 0$ for the generators α_1 of ${}_pG_{2p-3}$ and β_1 of ${}_pG_{2p(p-1)-2}$. The result of the present work, however, does not agree with this announcement. Our fundamental result is

Theorem. *For sufficiently large integer n, there exists a cell complex*

$$K=S^n\cup e^{n+2(p^2-1)(p-1)-1}\cup e^{n+2p^2(p-1)-1}\cup e^{n+2p^2(p-1)}$$

such that $\mathfrak{P}^{p^2}H^n(K;Z_p)\neq 0$, $\Delta\mathfrak{P}^1H^{n+2(p^2-1)(p-1)-1}(K;Z_p)\neq 0$ *and the cell* $e^{n+2(p^2-1)(p-1)-1}$ *is attached to* S^n *by a representative of* β_1^p.

It follows immediately the following

Corollary. $\alpha_1\beta_1^p=0$.

This shows that, in the Adams' spectral sequence computed by May [2], the differential cancells h_0b^p with b_1, or equivalently, the element γ of ${}_pG_{2p^2(p-1)-2}$ does not exist and should be cancelled with $\alpha_1\beta_1^p$. Then the corrected results for ${}_pG_k$ are stated as follows:

Proposition 1. *For $k<2(p^2+2p)(p-1)-4$, ${}_pG_k$ is the direct sum of cyclic groups generated by the following elements of corresponding degree k:*

$\alpha_i(1\leq i<p^2+2p,\ i\not\equiv 0(\mathrm{mod}\ p))$, $\alpha'_{jp}(1\leq j<p+2,\ j\not\equiv 0(\mathrm{mod\ p}))$,

α''_{p^2}, $\beta_1^r(1\leq r<p+3)$, $\alpha_1\beta_1^r(1\leq r<p)$,

$\beta_1^r\beta_s$, $\alpha_1\beta_1^r\beta_s(0\leq r,\ 2\leq s<p,\ r+s<p+2)$, $\beta_2\beta_{p-1}$, $\alpha_1\beta_2\beta_{p-1}$,

$\varepsilon_i(1\leq i<p)$, $\alpha_1\varepsilon_i(1\leq i<p-2)$, ε', $\beta_1\varepsilon'$, φ,

where $\deg(\alpha_i)=2i(p-1)-1$, $\deg(\alpha'_{jp})=2jp(p-1)-1$, $\deg(\alpha''_{p^2})=2p^2(p-1)-1$, $\deg(\beta_s)=2(sp+s-1)(p-1)-2$, $\deg(\varepsilon_i)=2(p^2+i)(p-1)-2$, $\deg(\varepsilon')=2(p^2+1)(p-1)-3$, $\deg(\varphi)=2(p^2+p)(p-1)-3$. *The orders of α'_{jp}, α''_{p^2}* and *φ are p^2, p^3, and p^2 respectively, and the other generators are of order p.* We mention that ε' corresponds to $\alpha_1\gamma$ in [2], $\alpha_1\varepsilon_i$ to $\alpha_{i+1}\gamma$, and $\beta_1\varepsilon'$ to $\alpha_1\beta_1\gamma$. The following representations of new generators are given:

$$\varepsilon'=\{\beta_1^p,\alpha_1,\alpha_1\},\quad \varepsilon_1=\{\alpha_1,\beta_1^p,p\iota,\alpha_1\},$$

$$\varepsilon_{i+1}=\{\varepsilon_i,p\iota,\alpha_1\},\quad 1\leq i<p-1,\quad \varphi\in\{\varepsilon_{p-2},\alpha_1,\alpha_1\}.$$