211. *A Product Theorem Concerning Some Generalized Compactness Properties*[1)]

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1. **Introduction.** In the last forty years a number of product theorems concerning compact topological spaces have been proven. In particular, Tychonoff [9] showed that the product of two compact spaces is a compact space, Dieudonné [1] showed that the product of a compact space and a paracompact space is a paracompact space, and Dowker [2] showed that the product of a compact space and a countably paracompact space is a countably paracompact space. These are three of the best-known theorems of the type: If X is a compact topological space and Y is a topological space with some generalized compactness property π, then the product space $X\times Y$ has the property π. The purpose of this paper is to prove a general theorem of this type and also to offer a unified approach to many generalized compactness properties.

2. **A characterization of some generalized compactness properties.** For each topological space X, let $\mathfrak{P}(X)$ be the set of all subsets of X. Let $\mathfrak{X}$ be the class of all topological spaces, let $\mathfrak{S}=\cup\{\mathfrak{P}\mathfrak{P}\mathfrak{P}(X): X\in\mathfrak{X}\}$ and let $Q:\mathfrak{X}\to\mathfrak{S}$ be a function with $Q(X)\in\mathfrak{P}\mathfrak{P}\mathfrak{P}(X)$ whenever $X\in\mathfrak{X}$.

Definition 1. *Q is* slattable *over X if and only if, whenever Y is a topological space and $\Lambda\in Q(X)$, there exists $\Gamma\in Q(X\times Y)$ such that whenever $G\in\Gamma$, then $G\subset L\times Y$ for some $L\in\Lambda$.*

Definition 2. *If Q is slattable over every topological space and m and n are infinite cardinals with $n\leq m$, then Q_n (respectively Q_n^m) is the class of all topological spaces X such that, if $\mathfrak{C}$ is an open cover of X ($\mathfrak{C}$ is an open cover of X with card $(\mathfrak{C})\leqslant m$), then there exists an open refinement $\mathfrak{R}$ of $\mathfrak{C}$ and $\Gamma\in Q(X)$ with each element of Γ intersecting fewer than n elements of $\mathfrak{R}$.*

Definition 3. *The functions C, P, and M from $\mathfrak{X}$ into $\mathfrak{S}$ are defined by:*

$$C(X)=\{\{X\}\}$$
$$P(X)=\{\mathfrak{C}:\ \mathfrak{C}\text{ is an open cover of }X\}$$
$$M(X)=\{\{\{x\}: x\in X\}\}.$$

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