209. *Note on Inverse Images under Open Finite-to-One Mappings*

By Akihiro OKUYAMA
Osaka Kyoiku University

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1. Introduction and theorems. Recently, A. Arhangel'skii [2] proved the following result:

A completely regular T_2 space which is the inverse image of a metric space under an open-closed finite-to-one mapping[1] *is metrizable.* Also, in the same paper he showed that *the inverse image of a compact metric space under an open finite-to-one mapping needs not be metrizable.*[2]

Hence, we shall consider the metrizability of it adding some assumptions and obtain the following result:

Theorem 1. *If f is an open finite-to-one mapping of a normal, locally compact T_2 space X onto a metric space Y, then X is metrizable.*

On the other hand, in [8] we introduced and discussed the notion of spaces with σ-locally finite nets[3] as a class of topological spaces containing all metric spaces. As for the space with a σ-locally finite net, the following holds:

Theorem 2. *Let f be an open finite-to-one mapping of a normal T_2 space X onto a collectionwise normal T_2 space with a σ-locally finite net. Then X has a σ-locally finite net.*

If we combine Theorem 2 with the notion of M-space (cf. [7]), we can obtain the another proof of the above Arhangel'skii's theorem and a generalization of it:

Theorem 3. *Let f be an open finite-to-one mapping of a normal T_2 space X onto a collectionwise normal T_2 space Y with a σ-locally finite net and g a closed mapping of X onto a metric space Z such that $g^{-1}(z)$ is countably compact for each $z \in Z$. Then X is metrizable.*

In the following we shall prove Theorems 2, 1, and 3 using some lemmas, and construct an example of *a non-metrizable hereditarily*

1) In this note we consider only continuous mapping.

2) The description of his example seems to contain some inaccuracies.

3) A collection $\mathfrak{B}$ of (not necessarily open) sets of a topological space X is called a *net* for X if, whenever $x \in U$ with x a point and U open in X, then $x \in B \subset U$ for some $B \in \mathfrak{B}$ (cf. [6], [3]). A net which is a union of countably many locally finite collections is called a *σ-locally finite net* (cf. [8]).