206. On the Sets of Points in the Ranked Space

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1. In this section we will define the several notions, m-open sets, m-closed sets and m-accumulation points of a subset in the ranked space [1], and we will prove some propositions in respect of these notions. We have used the same terminology as that introduced in the paper "On an Equivalence of Convergences in Ranked Spaces" [6].

Definition 1. *A subset A of a ranked space R is m-open if and only if for any point p of A there is a neighborhood $V_a(p)$ of p with a rank γ_a such that $V_a(p) \subseteq A$. A subset A is m-closed if and only if $R-A$ is m-open.*

Definition 2. *A point p is a m-accumulation point of a subset A of a ranked space R if and only if every neighborhood of p with any rank contains points of A other than p.*

Proposition 1. *If R is a ranked space, then the following conditions are equivalent.*

(*a*) *A subset A of R is m-closed.*

(*b*) *A subset A of R contains the set consisting of its m-accumulation points.*

Proof. To prove that (a) implies (b).

Let p be a m-accumulation point of A. If $p \notin A$ then $p \in R-A$. Since A is m-closed $R-A$ is m-open. Therefore there is some neighborhood $U(p)$ of p with a rank such that $U(p) \subseteq R-A$. Hence p is not a m-accumulation point of A.

To prove that (b) implies (a).

If A is not m-closed then $R-A$ is not m-open. Therefore there is a point p belonging to $R-A$ such that every neighborhood of p with any rank intersects A. Hence p is a m-accumulation point of A and does not belong to A.

Proposition 2. *If R is a ranked space, then the conditions below are related as follows. For all space* (*a*) *implies* (*b*). *If R satisfies the following condition* (*M*):

(*M*) *if $V(p) \in \mathfrak{V}_\alpha$, $U(p) \in \mathfrak{V}_\beta$, and $\alpha \geq \beta$ then $V(p) \subseteq U(p)$,*
then (*b*) *implies* (*a*).

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