7. *On Connections of Geometric Structures*

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Let G_0 and $\tilde{G}$ be Lie groups and $\rho: \tilde{G} \to G_0$ be a homomorphism. G_0 acts on another Lie group K from the left distributively:

$$a\cdot(k_1\cdot k_2)=(a\cdot k_1)\cdot(a\cdot k_2) \text{ for } a\in G_0 \text{ and } k_1, k_2\in K.$$

Let $\Theta: \tilde{G}\to K$ be a C^∞-mapping such that

$$(1) \qquad \Theta(a\cdot b)=\{\rho(b^{-1})\cdot\Theta(a)\}\cdot\Theta(b).$$

Then clearly $G=\{a\in G: \Theta(a)=1\}$ is a closed subgroup of $\tilde{G}$ and we have the

Proposition 1. *There is a canonical action of $\tilde{G}$ on K from the right, defined by*

$$(2) \qquad k\cdot a=\{\rho(a^{-1})\cdot k\}\Theta(a), \textit{ where } k\in K \textit{ and } a\in\tilde{G}.$$

Assume that $P(M, \tilde{G})$ be a C^∞-differentiable principal fibre bundle over a C^∞-manifold M of n dimensions. The we have two induced fibre bundles $T(M, K, \tilde{G})$ and $B(M, K, \tilde{G})$ over M with fibre K, associated with $P(M, \tilde{G})$, determined by ρ and the action of $\tilde{G}$ on K in Proposition 1, respectively. A C^∞-cross-section of $T(M, K, \tilde{G})$ is called, by the abuse of language, as a tensor field on M of type ρ, while we define a *connection of type* $(P(M, \tilde{G}), \rho, \Theta)$ as a C^∞-cross-section ω of $B(M, K, \tilde{G})$.

Proposition 2. *If ω_1 and ω_2 are two connections of type $(P(M, \tilde{G}), \rho, \Theta)$, then $\omega_1\cdot\omega_2^{-1}$ is a tensor field on M of type ρ.*
It must be remarked that in the above proposition $\omega_1^{-1}\cdot\omega_2$ is not necessarily a tensor field of type ρ, unless K is abelian.

Our definition generalizes that of Gunning [2], who studied the case where K is a vector space, $G_0=GL(K)$ and $P(M, \tilde{G})$ is $F^r(M)$ we define below.

Proposition 3. *The definition of the connection above includes those of principal fibre bundles of Ehresmann* [1], *of vector bundles as splittings of short exact sequences* (cf. P. Libermann [6]), *and of the bundles of higher order defined by Ehresmann* (cf. N. V. Quê [7]), *provided that $P(M, \tilde{G})$, ρ, Θ are suitably chosen.*

The proof is easily checked in all cases.

In the applications important is the following

Proposition 4. *If M is paracompact and K is a connected nilpotent Lie group, then there is a connection of type $(P(M, \tilde{G})\ \rho, \Theta)$.*

In the following we consider affine connections of higher order, as an example. Let $F^r(M)$ (resp. $F^{(r)}(M)$) be the set of all invertible