6. *Unions of Strongly Paracompact Spaces. II*[1)]

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As is well known, the space that is the union of two closed strongly paracompact[2)] subspaces need not be strongly paracompact (see [6]). In the previous note (see [8]), we have proved the following theorem:

Theorem 1. *Let $\mathfrak{F}=\{F_\alpha \mid \alpha \in A\}$ be a locally finite closed covering of a regular T_1-space X such that $Fr(F_\alpha)$*[3)] *has the locally Lindelöf property for any $\alpha \in A$. Then a necessary and sufficient condition that X be strongly paracompact is that F_α is strongly paracompact for any $\alpha \in A$.*

A main purpose of this note is to show the following theorem:

Theorem 2. *Let X be a normal T_1-space and $\mathfrak{G}=\{G_\alpha \mid \alpha \in A\}$ be a locally finite open covering of X such that $G_\alpha \cap G_\beta$ has the locally Lindelöf property with respect to its relative topology for each $\alpha, \beta \in A$* with $\alpha \neq \beta$. If *G_α is strongly paracompact for each $\alpha \in A$, then X is strongly paracompact.*

Proof. Suppose that A is well ordered. As is well known ([2]; Proposition 1.2), we can take the open covering $\mathfrak{H}=\{H_\alpha \mid \alpha \in A\}$ of X such that $\bar{H}_\alpha$[4)] $\subset G_\alpha$ for each $\alpha \in A$ and therefore $\bar{\mathfrak{H}}$[5)] is a locally finite closed covering of X. By the way to make the covering $\mathfrak{H}$,

$$X-\Big(\bigcup_{\alpha<\alpha_0} H\Big) \cup \Big(\bigcup_{\alpha>\alpha_0} G_\alpha\Big) \subset H_{\alpha_0} \subset \bar{H}_{\alpha_0} \subset G_{\alpha_0},$$

and then

$$Fr(\bar{H}_{\alpha_0}) \subset \bar{H}_{\alpha_0} - H_{\alpha_0} \subseteq G_{\alpha_0} - \Big[X - \Big\{\Big(\bigcup_{\alpha<\alpha_0} H\Big) \cup \Big(\bigcup_{\alpha>\alpha_0} G\Big)\Big\}\Big]$$
$$\subset G_{\alpha_0} \cap \Big(\bigcup_{\alpha\neq\alpha_0} G_\alpha\Big) = \bigcup_{\alpha\neq\alpha_0} (G_{\alpha_0} \cap G_\alpha),$$

then $\bigcup_{\alpha\neq\alpha_0} (G_{\alpha_0} \cap G_\alpha)$ has the locally Lindelöf property and hence $Fr(\bar{H}_{\alpha_0})$ has the locally Lindelöf property. After all we have the locally finite closed covering $\bar{\mathfrak{H}}=\{\bar{H}_{\alpha_0} \mid \alpha \in A\}$ such that $Fr(\bar{H}_{\alpha_0})$ has the locally Lindelöf property for any $\alpha \in A$. Therefore, by Theorem 1, we can

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1) This note is a continuation of the previous note [8].

2) The Hausdorff space X is *strongly paracompact* if an arbitrary open covering of X has the star finite open covering of X as a refinement.

3) $Fr(F_\alpha)$ denotes the boundary of $\bar{F}_\alpha$ in X, that is, $Fr(F_\alpha)=\bar{F}_\alpha \cap \overline{X-A_\alpha}$.

4) For the subset H_α of a topological space X, $\bar{H}_\alpha$ denotes the closure of H_α in X.

5) For the collection $\mathfrak{H}$ of subsets of a topological space X, $\bar{\mathfrak{H}}$ denotes the collection $\{\bar{U} \mid U \in \mathfrak{H}\}$.