4. *Relations between Unitary ρ-Dilatations and Two Norms*

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(Comm. by Kinjirô KUNUGI, M.J.A., Jan. 12, 1968)

Introduction. In this paper we discuss classes of power bounded operators on a Hilbert space H and we use the notations and terminologies of [5]. Following [1] [2] [5], an operator T on H possesses a unitary ρ-dilatation if there exists a Hilbert space K containing H as a subspace, a positive constant ρ and a unitary operator U on K satisfying the following representation

$$(1) \qquad T^n = \rho \cdot PU^n \qquad (n=1, 2, \cdots)$$

where P is the orthogonal projection of K on H. Put C_ρ the class of operators, whose powers T^n admit a representation (1).

It is well known that $T \in C_1$ is characterized by $\| T \| \leqq 1$. Moreover $T \in C_2$ is characterized by $\| T \|_N \leqq 1$, where $\| T \|_N$, usually called the numerical radius of T, is defined by

$$\| T \|_N = \sup | (Th, h) | \quad \textit{for every unit vector } h \textit{ in } H.$$

The latter fact was discovered by C.A. Berger (not yet published).

Using function theoretic methods, B. Sz-Nagy and C. Foias have given a characterization of C_ρ and shown the monotonity of C_ρ as a generalization of C_1 and C_2. Hence we may naturally expect that the condition for $T \in C_\rho$ depends upon $\| T \|$ and $\| T \|_N$ together. In this paper, as a continuation of calculations in the preceding paper [3], we give a simple sufficient condition for $T \in C_\rho$ related to both $\| T \|$ and $\| T \|_N$ and its graphic expression.

1. The following theorems are known.

Theorem A ([5]). *An operator T in H belongs to the class C_ρ if and only if it satisfies the following conditions:*

$$(\mathrm{i}) \quad \begin{cases} (I_\rho) \quad \| h \|^2 - 2\left(1-\dfrac{1}{\rho}\right)\mathrm{Re}(zTh, h) + \left(1-\dfrac{2}{\rho}\right)\| zTh \|^2 \geqq 0 \\ \qquad\qquad \textit{for } h \textit{ in } H \textit{ and } |z| \geqq 1, \\ (II) \quad \textit{the spectrum of } T \textit{ lies in the closed unit disk.} \end{cases}$$

(ii) *If $\rho \leqq 2$, then the conditon (I_ρ) implies (II).*

Theorem B ([5]). *C_ρ is non-decreasing with respect to the index ρ in the sense that*

$$C_{\rho_1} \subset C_{\rho_2} \qquad \textit{if } 0 \leqq \rho_1 < \rho_2.$$

Theorem C ([1]).