19. *Remarks on Bounded Sets in Linear Ranked Spaces*

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One of the authors defined the boundedness in linear ranked spaces ([2], and [3] p. 590).

Definition 1. A subset B in a linear ranked space is called *bounded* if, for any non-negative integer n, there is an integer $m (m \geq n)$ and a neighbourhood V of the origin and of rank m which absorbs B.

In the first half of this note, we shall study some of their properties, and in the latter half, examine the definition of bounded sets.

Throughout this note, "*linear ranked space*" will mean a linear space over the real or complex field, where are defined families $\mathfrak{V}_n (n=0, 1, 2, \cdots)$ of circled subsets satisfying the axioms (A), (B), (a), (b), (1), (2), and (3) in the note [2].

§ 1. Some properties. We shall set two problems.

(I) *Is the r-closure*[3)] *of any bounded set also bounded?*

(II) *Let A be an unbounded set. Can we choose a countable sequence of points of A having no bounded subsequence?*

In general, their answers are all negative. We shall show it and give some conditions which make them positive.

About problem I: Example 1. (*The counter of* (I)) Let E be the linear space of all bounded real valued functions on real line. (Addition and scalar multiplication are usual.) We define the sets

$$V(k, n)=\left\{\varphi(t) \in E \,\middle|\, |t|>k \Rightarrow |\varphi(t)|<\frac{1}{n}\right\}$$

$$k, n=0, 1, 2, \cdots; \frac{1}{0}=+\infty.$$

The families $\mathfrak{V}_n=\{V(k, n) \mid k=0, 1, 2, \cdots\}$ $(n=0, 1, 2, \cdots)$ possess the properties (A), (B), (a), (b), (1), (2), and (3) in the note [2], so E becomes a linear ranked space with indicator ω_0.

The set $B=V(1, 1)$ is bounded since, for any non-negative integer n, $\frac{1}{n+1} B \subseteq V(1, n)$. The r-closure $\mathrm{cl}(B)$ of B consists of all $\varphi(t)$

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3) For any subset A of a ranked space, the set of all points, each of which is an r-limit point of a countable sequence of points of A, is called the r-closure of A and denoted by $\mathrm{cl}(A)$.