12. *Note on the Nuclearity of Some Function Spaces.* I

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The definition of nuclearity in a general locally convex space was first given by A. Grothendieck [4]. The definition of nuclearity given by M. Gelfand and N. Ya. Vilenkin [3] concides with that of [4] in the case of countably normed spaces.

In this note, we consider the condition for nuclearity in A. Pietsch [6], which is mainly derived from A. Grothendieck. By using its condition, we shall show that $K_{\Omega}\{M_A\}$ space indroduced first by I. M. Gelfand and G. E. Shilov [2] and extended by T. Yamanaka [7] is nuclear.

1. Let E be a locally convex Hausdroff space over real or complex fields and U is any absorbent and absolutely convex neighborhood of the origin in E. Let

$$p_U(x)=\inf\{\rho>0;\ x\in\rho U\} \quad \text{for } x\in E$$

and

$$E_U=E/\{x\in E;\ p_U(x)=0\},$$

then topology of E_U is introduced by the norm

$$\|x_U\|=p_U(x) \quad \text{for } x_U\in E_U$$

where x_U coresponds to $x\in E$ in a natural way.

Let $C(M)$ be the sets of all continuous real or complex valued functions defined on M which is a compact Hausdroff space. Each continuous linear from μ on $C(M)$ is called *a Radon measure* on M and we frequently writes

$$\mu(f)=\int_M f d\mu.$$

A "*positive*" Radon measure is a $\mu\in C(M)'$ such that $\mu(f)\geqq 0$ whenever $f(x)\geqq 0$ for all $x\in M$.

Let E and F be normed spaces and their closed unit balls be U and V respectively. A continuous linear mapping T of E in F is called *nuclear mapping* if there exists continuous linear form $a_n\in E'$ and $y_n\in F$ such that the following holds:

$$Tx=\sum_N \langle x, a_n\rangle y_n \quad \text{for } x\in E$$

and

$$\sum_N P_{U^0}(a_n)P_V(y_n)<+\infty.$$

Definition. A locally convex Hausdroff space E be called *nuclear space* when there eixsts a base $\mathcal{U}(E)$ of absolutely convex, absorbent 0-neighborhood such that the following equivalent conditions holds:

i) for any $U\in\mathcal{U}(E)$ there exists a $V\in\mathcal{U}(E)$ being absorbed