27. Some Generalizations of QF-Rings

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1. Introduction. Throughout this paper all notations and all terminologies are the same as in T. Kato [5].

Recently there have been developed nice generalizations of *QF*-rings. B. L. Osofsky [6] has studied rings R for which R is an injective cogenerator in the category of right R-modules $\mathcal{M}_R$. Osofsky's theorem [6, Theorem 1] states that, if R is an injective cogenerator in $\mathcal{M}_R$, then R modulo its Jacobson radical J is Artinian. G. Azumaya [1] and Y. Utumi [8] have independently characterized rings R for which every faithful left R-module is a generator in ${}_R\mathcal{M}$. Such rings are called left *PF*. A theorem of Azumaya-Utumi states that a ring R is left *PF* if and only if R is left self-injective, R/J is Artinian, and every nonzero left ideal contains a simple one. T. Kato [4], [5] has studied rings R for which the injective hull $E(R_R)$ of R_R is torsionless and has proved the equivalence of the following statements:

(1) *R is right PF.*

(2) *R is an injective cogenerator in $\mathcal{M}_R$.*

(3) *$E(R_R)$ is torsionless and R is an S-ring.*

(4) *R is a cogenerator in $\mathcal{M}_R$ and is a right S-ring.*

In this paper we shall be concerned with the following condition:

(a) if U is a simple right (resp. left) ideal of a ring R, then there exists $a\in R$ such that $U\approx aR$, $E(aR)\subset R$ (resp. $U\approx Ra$, $E(Ra)\subset R$).

2. The condition (a). Proposition 1. *The following conditions on a ring R are equivalent*:

(1) *R satisfies* (a) *for simple right ideals.*

(2) *$E(U)$ is torsionless for each simple right ideal U.*

Proof. (1) implies (2) trivially.

(2) implies (1). Let U be a simple right ideal. Since $E(U)$ is torsionless by assumption, we have a map f: $E(U)\rightarrow R$ such that $U\rightarrow E(U)\rightarrow R$ is nonzero, or equivalently, a monomorphism by T. Kato [5, (1.1)]. f must be a monomorphism since $E(U)'\supset U$. From this our conclusion (1) follows immediately.

In my previous paper [5], we have discussed rings R for which $E(R_R)$ is torsionless. In the following we shall compare such rings