# 26. *Oscillatory Property of Certain Non-linear Ordinary Differential Equations*

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**1. Statement of theorems.** Recently, Kartsatos [2] proved that certain differential equations of the form

$$x''+f(t)g(x, x')=0 \quad \text{or} \quad x^{(2n)}+f(t)g(x)=0$$

can have only oscillatory solutions. Looking into the proofs in [2] closely, we see that the argument used there may be applied equally well to equations of the following more general form:

$$x^{(2n)}+f(t)g(x, x', \cdots, x^{(2n-1)})=0. \tag{1}$$

We shall prove in this paper the following theorems, where all functions considered are real-valued and continuous on their domains.

**Theorem 1.** *Suppose that the differential equation* (1) *satisfies the following hypotheses*:

($\alpha$) *$f$ is a positive function defined on the interval $I=[t_0, +\infty)$ with $t_0 \geqslant 0$ and $\int_{t_0}^{+\infty} f(t)dt=+\infty$;*

($\beta$) *$g$ is defined on $R^{2n}$; $\operatorname{sgn} g(x_1, x_2, \cdots, x_{2n})=\operatorname{sgn} x_1$ for any $(x_1, x_2, \cdots, x_{2n}) \in R^{2n}$; and $g(\lambda x_1, \lambda x_2, \cdots, \lambda x_{2n})=\lambda^{2p+1}g(x_1, x_2, \cdots, x_{2n})$ for any $(x_1, x_2, \cdots, x_{2n}) \in R^{2n}$, any $\lambda \in R$ and some non-negative integer $p$. Then, every solution of* (1) *on the interval $I$ is oscillatory.*

**Theorem 2.** *Suppose that the equation* (1) *satisfies* ($\alpha$) *and the following*:

($\gamma$) *$g$ is defined on $R^{2n}$; $\operatorname{sgn} g(x_1, x_2, \cdots, x_{2n})=\operatorname{sgn} x_1$ for any $(x_1, x_2, \cdots, x_{2n}) \in R^{2n}$; $g(-x_1, -x_2, \cdots, -x_{2n})=-g(x_1, x_2, \cdots, x_{2n})$ for any $(x_1, x_2, \cdots, x_{2n}) \in R^{2n}$; and for any $2 \leqslant k \leqslant 2n-1$ and any $c \geqslant 0$, the function $g(x_1, x_2, \cdots, x_{2n})$ has a definite limit $G(k, c)$, which is positive or $+\infty$, as $x_1 \to +\infty, \cdots, x_{k-1} \to +\infty, x_k \to c, x_{k+1} \to 0, \cdots, x_{2n} \to 0$. Then, every solution of* (1) *on $I$ is oscillatory.*

We would like to remark that Kartsatos [2] proved Theorem 1 in the case $n=1$ and Theorem 2 when the function $g$ depends only on the variable $x_1$.

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**2. Proof of theorems.** First we shall prove the following elementary but useful

**Lemma.** *Let $\varphi$ be a $2n$-times continuously differentiable func-*