57. On the Ranked Group

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(Comm. by Kinjirô KUNUGI, M. J. A., April 12, 1968)

The purpose of this note is to give a definition of the ranked group, i.e. to combine the notions of the group and the ranked space [1], by the same method in the definition of the topological group. Throughout this note, we shall treat only ranked spaces with indicator ω_0. We shall denote the points of a ranked space by $x, y, \cdots$, the family of neighbourhoods of x with rank n by $\mathfrak{V}_n(x)$, and fundamental sequences of neighbourhoods with respect to x[1] by $\{u_n(x)\}$, $\{v_n(x)\}, \cdots$.

§ 1. The definition of ranked groups. A set G is called a ranked group, if it is a group which is also a ranked space, where the group operations $(x, y)\to xy$, $x\to x^{-1}$, are continuous in the following sense;

(I) for any $\{u_n(x)\}$, $\{v_n(x)\}$, there exists a $\{w_n(xy)\}$ such that $u_n(x)v_n(y)\subseteq w_n(xy)$

(II) for any $\{u_n(x)\}$, there exists a $\{v_n(x^{-1})\}$ such that $(u_n(x))^{-1} \subseteq v_n(x^{-1})$.

(I) implies that, if $\{\lim x_n\}\ni x$ and $\{\lim y_n\}\ni y$, then $\{\lim x_n y_n\}\ni xy$, (II) implies that, if $\{\lim x_n\}\ni x$, then $\{\lim x_n^{-1}\}\ni x^{-1}$.

§ 2. The neighbourhoods of identity of a ranked group. Let G be a ranked group, and e be its identity. $\mathfrak{V}_n$ will denote the family of neighbourhoods of e with rank n, and $\{U_n\}, \{V_n\}, \cdots$ fundamental sequences of neighbourhoods with respect to e.

The system $\{\mathfrak{V}_n\}$ possesses the following properties:

(A) for every V in $\mathfrak{V}$, $e\in V$ (where $\mathfrak{V}=\bigcup_{n=0}^{\infty}\mathfrak{V}_n$)

(B) for any U, V in $\mathfrak{V}$, there is a W in $\mathfrak{V}$ such that $U\cap V\subseteq W$

(a) for any V in $\mathfrak{V}$ and for any integer n, there is an m, $m\geq n$, and a U in $\mathfrak{V}_m$ such that $U\subseteq V$

(b) $G\in\mathfrak{V}_0$.

These are obvious as the properties of neighbourhoods in a ranked space.

The axioms (I), (II) yields also,

(RG_1) for any $\{U_n\}$, $\{V_n\}$, there is a $\{W_n\}$ such that $U_nV_n\subseteq W_n$,

(RG_2) for any $\{U_n\}$, there is a $\{V_n\}$ such that $U_n^{-1}\subseteq V_n$,

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1) A sequence of neighbourhoods of x, $\{v_n(x)\}$, is called a fundamental sequence, if $v_n(x)\supseteq v_{n+1}(x)$, and $\alpha_n\uparrow\infty$, where α_n is the rank of $v_n(x)$.