53. On Theorems of Ontology

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We shall concern with a certain theorem having characteristic properties [1], [2].

In this paper we shall prove that the following expression is a theorem of ontology:

(α) $x \varepsilon X \equiv xa^*X \wedge [s]\{Sa^*x \supset xa^*S\}$.

The proof of (α) is based on the following only axiom of ontology given in 'S. Leśniewski's calculus of names' by J. Slupecki [2]:

$T1.$ $x \varepsilon X \equiv [\exists y]\{y \varepsilon x\} \wedge [y, z]\{y \varepsilon x \wedge z \varepsilon x \supset y \varepsilon z\} \wedge [y]\{y \varepsilon x \supset y \varepsilon X\}$.

The above axiom implies the following theorems:

$T2.$ $x \varepsilon X \wedge y \varepsilon x \supset x \varepsilon y$,

$T3.$ $x \varepsilon X \supset x \varepsilon V$,

$T4.$ $Sa^*P \supset SiP$,

$T5.$ $x \varepsilon V \supset (x \varepsilon S \equiv xa^*S)$,

$T6.$ $x \varepsilon X \equiv xiX \wedge \rightharpoondown/x/$.

In this system there are the following definitions:

$D1.$ $Sa^*P \equiv [\exists x]\{x \varepsilon S\} \wedge [x]\{x \varepsilon S \supset xP\}$,

$D2.$ $\rightharpoondown/x/ = [y, z]\{y \varepsilon x \wedge z \varepsilon x \supset y \varepsilon z\}$.

The proofs of theorems will be given in the form of suppositional proofs used by J. Slupecki.

(I) $xa^*X \equiv [\exists y]\{y \varepsilon x\} \wedge [y]\{y \varepsilon x \supset y \varepsilon X\}$. $\{D1\}$

(II) $[S]\{Sa^*x \supset xa^*S\} \wedge y \varepsilon x \supset xa^*y$.

Proof.	(1)	$[S]\{Sa^*x \supset xa^*S\}$	{premises}
	(2)	$y \varepsilon x$	
	(3)	$y \varepsilon V$	$\{T3, 2\}$
	(4)	ya^*x	$\{T5, 2, 3\}$
	(5)	$ya^*x \supset xa^*y$	$\{O\Pi : 1\}$
		xa^*y	$\{5, 4\}$

(III) $[S]\{Sa^*x \supset xa^*S\} \wedge y \varepsilon x \wedge z \varepsilon x \supset y \varepsilon z$.

Proof.	(1)	$[S]\{Sa^*x \supset xa^*S\}$	{premises}
	(2)	$y \varepsilon x$	
	(3)	$z \varepsilon x$	
	(4)	xa^*y	$\{II, 1, 2\}$
	(5)	$[z]\{z \varepsilon x \supset z \varepsilon y\}$	$\{D1, 4\}$
	(6)	$z \varepsilon x \supset z \varepsilon y$	$\{O\Pi : 5\}$
	(7)	$z \varepsilon y$	$\{6, 3\}$
		$y \varepsilon z$	$\{T2, 3, 7\}$