46. *Extended p-th Powers of Complexes and Applications to Homotopy Theory*

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(Comm. by Zyoiti SUETUNA, M. J. A., April 12, 1968)

1. Extended p-th power of a complex. Throughout this note p will denote an odd prime, $m=(p-1)/2$, $\pi=Z_p$ a cyclic group of order p, and the homology and cohomology groups will have the coefficient group Z_p. Let $W=W^\infty(=S^\infty)$ be a regular π-free acyclic CW-complex having one π-free basic cell e_i for each dimension i. The cells e_i are oriented such that in the infinite dimensional lens space W/π the dual $w_i \in H^i(W/\pi)$ of the class of e_i satisfies $w_{2i}=(w_i)^2$ and $\beta(w_2)=w_1$ for the cohomology Bockstein β.

For a finite CW-complex X, the product and the reduced join of p-copies of X will be denoted by $X^p=X\times\cdots\times X$ and $X^{(p)}=X\wedge\cdots\wedge X$ respectively. Let π acts on X^p and $X^{(p)}$ as cyclic permutations of the factors, and consider the quotient complexes

$$W^r\times_\pi X^p \quad \text{and} \quad ep^r(X)=(W^r\times_\pi X^{(p)})/(W^r/\pi),$$

where W^r indicates the r-skeleton of X and $W^r/\pi=W^r\times_\pi x_0^{(p)}$ for the base point x_0 of X. Let $x_0, x_1, x_2, \cdots$ be a Z_p-basis of homogeneous elements of $H_*(X)$ which satisfies that if $\Delta x_j\neq 0$ for the homology Bockstein then $\Delta x_j=x_l$ for some l. A Z_p-basis of $H_*(W\times_\pi X^p)$ is given as the classes represented by the following cycles (cf. [2], [3]):

$$e_i\otimes_\pi x_j^p,\quad j=0,1,2,\cdots,\quad x_j^p=x_j\otimes\cdots\otimes x_j \ (p\text{-times}),$$
$$e_0\otimes_\pi(x_{j_1}\otimes\cdots\otimes x_{j_p}),\quad j_s\neq j_t \ \text{ for some } s,t,$$

where $(j_1,\cdots,j_p)$ runs through each representatives of the classes obtained by cyclic permutations of the indices. The same result holds for $H_*(W^r\times_\pi X^p)$ restricting e_i by $0\leq i\leq r$ and by adding cycles of the form $\partial(e_{r+1}\otimes_\pi(x_{j_1}\otimes\cdots\otimes x_{j_p}))$.

By the natural projection $W^r\times_\pi X^p\to ep^r(X)$, a Z_p-basis of $\tilde{H}_*(ep^r(X))$ is obtained from that of $H_*(W^r\times_\pi X^p)$ by omitting the cycles containing x_0.

Denote by $P^i_*: H_q\to H_{q-2i(p-1)}$ the dual of the Steenrod reduced power P^i, and let $P^i_*x_k=\Sigma_j a_{k,j}(i)x_j$ for $a_{k,j}\in Z_p$. Then the following relation has been established in [3].

Theorem 1. (*Nishida*).

$$P^n_*(e_{c+2n(p-1)}\otimes_\pi x_k^p)=\Sigma_{i,j}\binom{[c/2]+qm}{n-pi}a_{k,j}(i)(e_{c+2ip(p-1)}\otimes_\pi x_j^p)$$