80. On Nuclear Spaces with Fundamental System of Bounded Sets. I

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We consider a nuclear space with a fundamental system of bounded sets. In this paper, we consider the open mapping and closed graph theorems in the nuclear space.

For nuclear spaces and its related notion see [8]. Most of the definitions and notations of locally convex spaces are taken from N. Bourbaki [1] and T. Husain [4].

1. In this section, we consider under what conditions nuclear space is the space with the open mapping and closed graph theorems.

Definition 1. *Let E be a locally convex vector space and E' its dual.*

(1) *ew^*-topology is defined to be the finest topology on E' which coincides with $\sigma(E', E)$ on each equicontinuous set of E'.*

(2) *p-topology is the $\mathfrak{S}$-topology on E' where $\mathfrak{S}$ consists of all precompact subsets of E.*

(3) *E is called a S-space if on E', $ew^*=p$.*

(4) *E is called a B-complete if a linear continuous and almost open mapping of E onto any locally convex vector space F is open.*

(5) *E is called a $B(\mathfrak{T})$-space if it satisfies the following statement; For each barrelled space F, a linear and continuous mapping of E onto F is open.*

Let E and F be normed spaces, U and V their closed unit balls respectively. A continuous linear mapping T of E in F is called a nuclear mapping if there exists a continuous linear form $a_n \in E'$ and $y_n \in F$ such that the following holds;

$$Tx=\sum_N <x, a_n> y_n \quad \text{for} \quad x \in E$$

and

$$\sum_N P_{U^0}(a_n) P_V(y_n) < +\infty.$$

For each nuclear mapping T define the norm:

$$\nu(T)=\inf\{\sum_N P_{U^0}(a_n)P_V(y_n)\}.$$

Let $\mathfrak{N}(E, F)$ be the set of all nuclear mappings of E into F, we introduce a norm in $\mathfrak{N}(E, F)$ by $\nu(T)$. Let $\mathcal{L}(E, F)$ be the set of all continuous linear mappings of E into F, let $\mathcal{A}(E, F)$ be the set of all mappings $t \in \mathcal{L}(E, F)$ such that $t(E)$ is a finite dimentional subspace