76. On the Compatibility of the AP- and the D-integrals

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(Comm. by Kinjirô KUNUGI, M. J. A., May 13, 1968)

1. Introduction. We call two definitions of integration *compatible* if every function which is integrable in both senses is integrable to the same value in both senses. H. W. Ellis [3] has shown that the AP-integral and the CP-integral [2] are not compatible. Recently V. A. Skvorcov [5] established that, if $f(x)$ is CP-integrable with the CP-integral $F(x)$ as well as D-integrable with the D-integral $F_1(x)$ over $[a, b]$ then $F_1(x)=F(x)+C$ on $[a, b]$ where C is a constant. This assertion shows that the CP-integral do not contradict the general Denjoy integral.

The aim of this paper is to show *directly* that the D-integral and the AP-integral are compatible.

2. The AP-integral. A real valued function $f(x)$ is said to be $\underline{AC}$ on a linear set E if, to each positive number ε, there exists a number $\delta>0$ such that

$$\sum\{f(b_k)-f(a_k)\}>-\varepsilon$$

for all finite non-overlapping sequences of intervals $\{(a_k, b_k)\}$ with end points on E and such that

$$\sum(b_k-a_k)<\delta.$$

There is a corresponding definition $\overline{\mathrm{AC}}$ on E. A function which is both $\underline{AC}$ and $\overline{AC}$ on E is termed AC on E. If the set E is the sum of a countable number of sets E_k on each of which $f(x)$ is $\underline{AC}$ then $f(x)$ is said to be $\underline{ACG}$ on E. If the sets E_k are assumed to be closed, then $f(x)$ is said to be $(\underline{ACG})$ on E. Similarly we can define $\overline{ACG}$ and $(\overline{ACG})$ on E. A function is said to be (ACG) on E if it is both $(\underline{ACG})$ and $(\overline{ACG})$ on E. A continuous function which is both $\underline{ACG}$ and $\underline{ACG}$ on E is termed ACG on E.

The function $M(x)$ is called an *AP-major* function of $f(x)$ in [a, b] if

(i) $M(a)=0$;

(ii) $M(x)$ is approximately continuous for all $x\in[a, b]$;

(iii) $\underline{AD}\ M(x)\geqq f(x)$ everywhere on $[a, b]$;

(iv) $\underline{AD}\ M(x)>-\infty$ everywhere on $[a, b]$.

The *AP-minor* function $m(x)$ is defined analogously.