70. *Calculus in Ranked Vector Spaces. III*

By Masae YAMAGUCHI
Department of Mathematics, University of Hokkaido

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(1.7.6) It is obvious that one has the following; if $\{x_n\}$ is a quasi-bounded sequence and $\{a_n\}$ is a bounded sequence in $\mathfrak{R}$ (i.e., $|a_n|<M$, for $n=0, 1, 2, \cdots$), then $\{a_n x_n\}$ is also a quasi-bounded sequence.

In fact, let $\{\mu_n\}$ be a sequence in $\mathfrak{R}$ with $\mu_n \to 0$, then

$$\mu_n(a_n x_n)=(\mu_n a_n)x_n.$$

Since $|a_n|<M$, $\mu_n a_n \to 0$ in $\mathfrak{R}$. Using that $\{x_n\}$ is a quasi-bounded sequence, we have

$$\therefore \quad \{\lim \mu_n(a_n x_n)\} \ni 0.$$

(1.7.7) **Proposition.** *Let $l: E_1 \to E_2$ be a linear and continuous map between ranked vector spaces E_1, E_2. If $\{x_n\}$ is a quasi-bounded sequence in a ranked vector space E_1, then $\{l(x_n)\}$ is also a quasi-bounded sequence in E_2.*

Proof. Let $\{\mu_n\}$ be a sequence in $\mathfrak{R}$ such that $\mu_n \to 0$. Then it follows from the linearity of l that $\mu_n l(x_n)=l(\mu_n x_n)$. Using the assumption that $l: E_1 \to E_2$ is continuous,

$$\{\lim \mu_n l(x_n)\} \ni l(0)=0.$$

Therefore $\{l(x_n)\}$ is a quasi-bounded sequence.

(1.7.8) **Proposition.** *Let $E_1, E_2, \cdots, E_m$ be a family of ranked vector spaces. For a sequence $\{z_n\}=\{(x_{n1}, x_{n2}, \cdots, x_{nm})\}$ of the direct product $\times E_i$ to be a quasi-bounded sequence it is necessary and sufficient that, for each i $(i=1, 2, \cdots, m)$, $\{x_{ni}\}$ is a quasi-bounded sequence in E_i.*

Proof. Let $\{\mu_n\}$ be a sequence in $\mathfrak{R}$ with $\mu_n \to 0$. Then

$$\mu_n z_n=(\mu_n x_{n1}, \mu_n x_{n2}, \cdots, \mu_n x_{nm}).$$

By (1.5.1), $\{\lim \mu_n z_n\} \ni 0$ is equivalent to

$$\{\lim \mu_n x_{n1}\} \ni 0, \{\lim \mu_n x_{n2}\} \ni 0, \cdots, \{\lim \mu_n x_{nm}\} \ni 0.$$

That is, our assertion holds.

1.8. L-convergence. Let us introduce a new convergence in a ranked vector space E, where the convergence in the sense of (1.2.1) is defined.

(1.8.1) **Definition.** Let $\{x_n\}$ be a sequence of a ranked vector space E. We say that *a sequence $\{x_n\}$ converges to x in the sense of L-convergence*, and we write $\{\mathrm{Lim}\, x_n\} \ni x$ if and only if x_n can be