66. *On Tabooistic Treatment of Proposition Logics*

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(Comm. by Zyoiti SUETUNA, M.J.A., May 13, 1968)

1. The purpose of this short note is to remark that the tabooistic treatment of formal theories introduced in my paper [1] can be nicely applied to dealing with axiomatizable proposition logics which are stronger than or equivalent to the *generalized minimal proposition logic*. The *minimal proposition logic* ***LMS*** has $\rightarrow$ (implication), $\wedge$ (conjunction), $\vee$ (disjunction), and $\sim$ (negation) as its logical constants and is characterized by the following inference rules:

F: $\mathfrak{A}$ *is deducible from* $\mathfrak{A}$.

I: $\mathfrak{A}$ *is deducible from* $\mathfrak{B}$ *and* $\mathfrak{B}\rightarrow\mathfrak{A}$.

I*: $\mathfrak{A}\rightarrow\mathfrak{B}$ *is deducible from the fact that* $\mathfrak{B}$ *is deducible from* $\mathfrak{A}$.

C: $\mathfrak{A}$ *as well as* $\mathfrak{B}$ *is deducible from* $\mathfrak{A}\wedge\mathfrak{B}$.

C*: $\mathfrak{A}\wedge\mathfrak{B}$ *is deducible from* $\mathfrak{A}$ *and* $\mathfrak{B}$.

D: $\mathfrak{A}$ *is deducible from* $\mathfrak{B}\vee\mathfrak{C}$, $\mathfrak{B}\rightarrow\mathfrak{A}$, *and* $\mathfrak{C}\rightarrow\mathfrak{A}$.

D*: $\mathfrak{A}\vee\mathfrak{B}$ *is deducible from* $\mathfrak{A}$ *as well as from* $\mathfrak{B}$.

N: $\sim\mathfrak{A}$ *stands for* $\mathfrak{A}\rightarrow$ 人, *where* 人 *is a proposition constant.*

In *generalized formalism* of proposition logic where we adopt the universal quantification ranging over proposition variables $x, y, \cdots$, we can reformulate the minimal proposition logic as the logic ***LMS**** characterized by the following inference rules and axioms:

Inference rules: ***F***, ***I***, ***I****, and

$\bar{U}$: $\mathfrak{A}(\mathfrak{F})$ *is deducible from* $(x)\mathfrak{A}(x)$, *where* $\mathfrak{F}$ *is a propositional expression containing no quantification.*

Axioms:

c1: $(x)(y)(x\wedge y\rightarrow x)$, **c2**: $(x)(y)(x\wedge y\rightarrow y)$,

c*: $(x)(y)(x\rightarrow(y\rightarrow x\wedge y))$,

d: $(x)(y)(z)(y\vee z\rightarrow((y\rightarrow x)\rightarrow((z\rightarrow x)\rightarrow x)))$,

d*1: $(x)(y)(x\rightarrow x\vee y)$, **d*2**: $(x)(y)(y\rightarrow x\vee y)$,

n1: $(x)(\sim x\rightarrow(x\rightarrow$ 人$))$, **n2**: $(x)((x\rightarrow$ 人$)\rightarrow\sim x)$.

Any proposition $\mathfrak{A}$ containing no quantification is provable in ***LMS*** if and only if $\mathfrak{A}$ is provable in the *generalized minimal proposition logic* ***LMS****.

In generalizing the notion "intermediate proposition logic", I will call any proposition logic ***L***, in generalized formalism or not, an intermediate proposition logic if and only if every provable proposition in