111. A Limit Theorem of a Pulse-Like Wave Form for a Markov Process

By Masao NAGASAWA*)

Tokyo Institute of Technology and Stanford University

(Comm. by Kunihiko KODAIRA, M. J. A., June 12, 1968)

Nagumo, Arimoto, and Yoshizawa [3] discussed the asymptotic behavior of the solution of the following equation, which describes an active pulse transmission line simulating an animal nerve axon,

$$
\frac{\partial^3 u}{\partial t\,\partial x^2}=\frac{\partial^2 u}{\partial t^2}+\mu(1-u+\varepsilon u^2)\frac{\partial u}{\partial t}+u, \tag{1}
$$

$$
\mu>0,\quad \frac{3}{16}>\varepsilon>0,\quad x>0,\quad t>0,
$$

with the boundary data $u(0, x)=0\ (x\geq 0)$, $\frac{\partial u}{\partial t}(0, x)=0\ (x\geq 0)$, and $u(t, 0)=\psi(t)\ (t\geq 0)$, $\psi(t)\equiv 0$ for $t\geq t_0$. The equation (1) may be written as a system of equations

$$
\tag{1'}
\begin{cases}
\dfrac{\partial u}{\partial t}=\dfrac{\partial^2 u}{\partial x^2}-\mu\left(u-\dfrac{1}{2}u^2+\dfrac{\varepsilon}{3}u^3\right)-w \\
\dfrac{\partial w}{\partial t}=u.
\end{cases}
$$

They showed experimentally that the solution is a specific pulse-like wave form such that, when t increases, smaller signals are amplified, larger ones are attenuated, narrower ones are widen and those which are wider are shrunk, all approaching a specific wave form; and there is a threshold value to the signal height, and signals below the threshold (or noise) are eliminated when $t\to\infty$.

A probabilistic model for the Nagumo *et al.*'s equation was given in terms of a branching Markov process with age and sign in [2].[1] Since such a limiting property stated above is new in the theory of Markov processes, it is an attractive problem to discuss the limit theorem[2] in connection with the probability theory.[3] The objective

---

*) Research supported in part under grant NSF G. P. 7110 at Stanford University, Stanford, California.

1) For branching Markov processes with age and sign cf. also [5].

2) Yamaguchi proved some limit theorem for (1′) in [6].

3) H. P. McKean obtained some results of the problem for FitzHugh's equation, a version of (1′),

$$
\begin{cases}
\dfrac{1}{c}\dfrac{du}{dt}=\left(u-\dfrac{u^3}{3}\right)+w, \\
c\dfrac{dw}{dt}=a-u, \quad \text{(private communication).}
\end{cases}
$$