106. σ-Spaces and Closed Mappings. I

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1. Introduction. In our previous paper [7] we have introduced the notion of a σ-locally finite net as a generalization of a countable net (cf. [2], [5]) and studied the spaces with σ-locally finite nets as a class of a topological spaces which contains all metric spaces.

Definition. A collection $\mathfrak{B}$ of subsets of a topological space X is said to be a *net for* X if the following condition is satisfied:

For every point x of X and every open neighborhood U of x there exists an element B of $\mathfrak{B}$ with $x \in B \subset U$.

A collection $\mathfrak{B}$ of subsets of X is said to be a *σ-locally finite net* if it is a net and it is a union of a countable number of subcollections which are locally finite in X. We shall say that X is a *σ-space* if X has a σ-locally finite net (cf. [6]).

The notion of net was introduced and discussed by A. Arhangel'skii [1][1] and several results were obtained by him in [1], [2] and, also, by E. Michael [4] in the case of countable nets.

The purpose of this paper is to study the images of σ-spaces under closed mappings[2] and to prove the following two theorems.

Theorem 1. *Let f be a closed mapping of a normal T_1 σ-space X onto a topological space Y. Then the set $\{y \mid \partial f^{-1}(y)$ is not countably compact$\}$ is a σ-discrete subset of Y;* that is, *it is a countable union of discrete subsets of Y, where $\partial f^{-1}(y)$ denotes the boundary of $f^{-1}(y)$ for each $y \in Y$.*

Theorem 2. *Let f be a closed mapping of a normal T_1 σ-space X onto a topological space Y. Then Y is a σ-space, too.*

As regards Theorem 1 N. Lašnev [3] proved it in the case of metric space. He proved also, in another paper [4], the following theorem:

In order that a T_1 space X be a closed image of a metric space, it is necessary and sufficient that X is a Fréchet-Urysohn space[3] with

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1) This fact was pointed out to us by Professor A. Arhangel'skii. We express our thanks to his advice.

2) All mappings in this paper are *continuous*.

3) X is a *Fréchet-Urysohn space* if, for every subset M of X and $x_0 \in \overline{M}$, there exists a sequence $\{x_n \mid n=1, 2, \cdots\}$ of points of M, converging to x_0.