103. Integration with Respect to the Generalized Measure. IV

By Masahiro TAKAHASHI
Department of Economics, Osaka City University

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In this part of the paper, we discuss an integral in a concrete form—an integral 'of a (measurable) function f over a (measurable) set X' by a measure μ.

1. **Definition of an integral system.** *Let M be a non-empty set. Let G, K, and J be topological additive groups*[1] *and assume that, for each $g \in G$ and $k \in K$, the product $g \cdot k$ of g and k is defined as an element of J satisfying the conditions:*

1) $(g+g') \cdot k = g \cdot k + g' \cdot k$,

2) $g \cdot (k+k') = g \cdot k + g \cdot k'$,

for each $g, g' \in G$, and $k, k' \in K$.

Now let us denote by $\mathcal{F}$ the additive group of all K-valued functions defined on M (the sum of two functions in $\mathcal{F}$ is defined in the usual way). We consider $\mathcal{F}$ as a topological group, in which the family of all sets of the form $\{f \mid f \in F, f(M) \subset P\}$, where P is a neighbourhood of the unit element of K, constitutes a base of the system of neighbourhoods of the unit element of $\mathcal{F}$. This topology is characterized as the topology such that any sequence of elements of $\mathcal{F}$ converges in the space $\mathcal{F}$ if and only if the sequence uniformly converges as a functional sequence.

Then the map φ of K into $\mathcal{F}$ defined by $(\varphi(a))(x) = a$, for each $a \in K$ and $x \in M$, is an isomorphism of the topological group K into $\mathcal{F}$ so that we may identify the topological group K, by the isomorphism φ, with the subgroup $\varphi(K)$ of $\mathcal{F}$.

Let $\mathcal{M}$ be the family of all subsets of M. Then $\mathcal{M}$ is a ring (in the algebraic sense) of which each element is an idempotent, when we define, for each X and Y in $\mathcal{M}$, $X+Y$ and XY by $(X-Y) \cup (Y-X)$ and $X \cap Y$, respectively.

For each $X \in \mathcal{M}$ and $f \in \mathcal{F}$, denote Xf the function in $\mathcal{F}$ such that

$$(Xf)(x) = \begin{cases} f(x) & \text{if } x \in X, \\ 0 & \text{if } x \in M-X. \end{cases}$$

Then each element X of $\mathcal{M}$ is considered as a continuous homomorphism of $\mathcal{F}$ into itself satisfying the conditions:

1) The topology of G plays no role here.