101. On the Nörlund Summability of Fourier Series and its Conjugate Series

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§ 1. Let $\{p_n\}$ be a sequence such that $P_n=p_0+p_1+\cdots+p_n\neq 0$ for $n=0,1,2,\cdots$. A series $\sum_{n=0}^{\infty} a_n$ with its partial sum s_n is said to be summable (N, p_n) to sum s, if

$$\frac{1}{P_n}\sum_{k=0}^{n} p_{n-k}s_k \to s \quad \text{as} \quad n\to\infty.$$

Let $f(t)$ be a periodic finite-valued function with period 2π and integrable (L) over $(-\pi,\pi)$. Let its Fourier series be

$$\frac{1}{2}a_0+\sum_{n=1}^{\infty}(a_n\cos nt+b_n\sin nt)=\sum_{n=0}^{\infty}A_n(t). \tag{1.1}$$

Then the conjugate series of (1.1) is

$$\sum_{n=1}^{\infty}(b_n\cos nt-a_n\sin nt)=\sum_{n=1}^{\infty}B_n(t). \tag{1.2}$$

Throughout this paper, we write

$$\varphi(t)\equiv\frac{1}{2}\{f(x+t)+f(x-t)-2f(x)\}, \qquad \Phi(t)\equiv\int_0^t|\varphi(u)|\,du,$$

$$\psi(t)\equiv\frac{1}{2}\{f(x+t)-f(x-t)\}, \qquad \Psi(t)\equiv\int_0^t|\psi(u)|\,du$$

and $\tau=[1/t]$, where $[\lambda]$ is the integral part of λ.

The purpose of this paper is to prove the following two theorems.

Theorem 1. *Let $\{p_n\}$ be a sequence such that*

$$p_n>0,\ p_n\downarrow \quad \text{and} \quad P_n\to\infty. \tag{1.3}$$

And let $\lambda(t)$ be a positive integrable function such that

$$\int_\eta^n\frac{\lambda(u)}{u}\,du=O(P_n) \quad \text{as} \quad n\to\infty, \tag{1.4}$$

for any fixed $\eta>0$. If

$$\Phi(t)=o\left(t\lambda\left(\frac{1}{t}\right)/P_\tau\right) \quad \text{as} \quad t\to+0, \tag{1.5}$$

then the series $\sum_{n=0}^{\infty}A_n(x)$ is summable (N,p_n) to sum $f(x)$.

Theorem 2. *Let $\{p_n\}$ and $\lambda(t)$ be defined as in Theorem 1. If*

$$\Psi(t)=o\left(t\lambda\left(\frac{1}{t}\right)/P_\tau\right) \quad \text{as} \quad t\to+0, \tag{1.6}$$