150. Weak Convergence of the Isotropic Scattering Transport Process with One Speed in the Plane to Brownian Motion

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Let us consider a particle moving in the d-dimensional Euclidian space R^d. It travels in a straight line with constant speed and after some random time it undergoes scattering which changes its moving-direction and so, after scattering, continues to move as if it starts afresh. Now let $P(t, x, \Gamma)$ be the probability that we can find the particle in a region Γ at time t when it starts from x. In 2-dimensional isotropic case, Monin [4] has obtained the explicite formula of $P(t, x, \cdot)$ and has shown that it converges to 2-dimensional Gaussian distribution as $t\to\infty$. On the other hand, in one-dimensional isotropic case, Ikeda and Nomoto [3] have proved that $P(t, x, \cdot)$ converges to a Gaussian distribution as the speed of the particle tends to infinity in an appropriate manner and moreover they have shown that the measure on the space of trajectories of the motion also converges to the Wiener measure.

The purpose of this paper is to prove that the same result to them is also valid for the two-dimensional case.

1. Notations and definitions. Let $\Theta=[\theta(t), +\infty, \mathcal{N}_t, P.]$ be a right continuous jump process on the state space $[-\pi, \pi)$, which is identified to the unit circle S^1. Also let τ be the first jumping time of Θ, i.e., $\tau=\inf\{t:\theta(t)\neq\theta(0)\}$. We assume that the following conditions be satisfied:

(i) $P\{\tau>t\}=e^{-c^2t}$, $c>0$ constant,

(ii) $P\{\theta(\tau)\in\Gamma\}=|\Gamma|/2\pi$,

where $|\Gamma|$ denotes the Lebesgue measure of the set Γ.

The formula

$$A(t)=\left(\int_0^t\cos\theta(s)ds, \int_0^t\sin\theta(s)ds\right) \tag{1.1}$$

defines on R^2-valued continuous additive functional of Θ. Let $E=R^2\times S^1$ be the product space of R^2 and S^1, and $\mathcal{B}(E)$ be the topological Borel field of E. For each point $(x, \theta)\in E$, we define the following:

$$X^{(x,\theta)}(t)=(x+cA(t), \theta(t)), \tag{1.2}$$

$$P^{(x,\theta)}\{X^{(x,\theta)}(t)\in B\}=P\{X^{(x,\theta)}(t)\in B\}, \qquad B\in\mathcal{B}(E), \tag{1.3}$$

$$\mathcal{M}_t^{(x,\theta)}=\mathcal{N}_t. \tag{1.4}$$