149. *Approximation of Transport Process by Transport Chain*

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It is classical but important that the difference equations are closely related to diffusional equations. In connection with stochastic problems, this fact shows that discrete models serve as an approximation to continuous models of random motions. As is well known, the Brownian motion appears as a limit of random walk in various senses. In particular, F. Knight [3] has made pathwise approximation of Brownian motion by joining paths of random walk.

Now, let us consider the telegraph equation of infinite cable. Then it will be seen that there corresponds a stochastic process, called the transport process.

The purpose of this paper is to construct the approximate discrete chain of transport process (Theorem 1) and next, to prove, similarly to the Knight's result in Brownian motion's case, the pathwise convergence of this discrete chain to the transport process (Theorem 2).

1. Definition. Let S be the product space of one-dimensional Euclidian space E^1 and the two points set $\Theta=\{\theta=\pm 1\}$. Let $X=[X(t)=(x(t), \theta(t)), +\infty, \mathcal{M}_t, P_{(x,\theta)}, (x, \theta)\in S]$ be the right continuous strong Markov process over the state space S such that

(i) $P_{(x,\theta)}\{X(t)=(x+c\theta t, \theta)\mid t<\tau\}=1$,
where $\tau=\inf\{t:\theta(t)\neq\theta(0)\}$,

(ii) $P_{(x,\theta)}\{\tau>t\}=e^{-\kappa t}$,

(iii) $P_{(x,\theta)}\{X(\tau)=(y, -\theta')\mid X(\tau-)=(y,\theta')\}=1$.

Definition 1.1. The Markov process X is called *the transport process* (with speed c).

For simplicity, we always suppose that

Assumption 1.1. $c=1$, $\kappa=1$.

Let us denote by $\{T_t\}$ the semigroup corresponding to the transport process X, i.e.

$$T_t f(x,\theta)=E_{(x,\theta)}[f(X(t))], \tag{1.1}$$

where $E_{(x,\theta)}$ is the expectation with respect to $P_{(x,\theta)}$-measure.

Proposition 1.1. *For any given nice function f on S, $U(t,x,\theta)=T_t f(x,\theta)$ is the unique solution of the following telegraph equation:*

$$\begin{cases} \dfrac{\partial}{\partial t}U(t,x,\theta)=\theta\dfrac{\partial}{\partial x}U(t,x,\theta)-U(t,x,\theta)+U(t,x,-\theta), \\ U(t,x,\theta)\to f(x,\theta) \quad as \quad t\to 0. \end{cases} \tag{1.2}$$