# *148. The Completion of a Convergence Space in the Sense of H. R. Fisher*

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In this paper we shall make a study of the completion of a space: here by a space we mean a set in which there is defined a closure operation satisfying three conditions $A\subseteq\overline{A}$, $\overline{\phi}=\phi$, and $\overline{A\cup B}=\overline{A}\cup\overline{B}$.

Such a space was introduced by Tukey [8] and studied also by Fisher [4] under the name of a convergence space.[1]

In this paper we shall describe a space by assigning a neighborhood system to each point of it.

Thus we get a generalization of the results of the author's paper [7].[2]

§ 1. Let $\varphi$ be a mapping of a set $X$ into a set $Y$. Then for a family $\mathfrak{A}$ consisting of subsets of $X$, we will denote by $\varphi(\mathfrak{A})$ the family $\{\varphi(A)\,|\,A\in\mathfrak{A}\}$ and for a family $\mathfrak{B}$ consisting of subsets of $Y$, let's denote by $\varphi^{-1}(\mathfrak{B})$ the family $\{\varphi^{-1}(B)\,|\,B\in\mathfrak{B}\}$.

Let $X$ be a subset of a set $X^*$, then for a filter $\mathfrak{f}$ in $X$, the filter in $X^*$ generated by $\mathfrak{f}$ is denoted by $\mathfrak{f}^*$.

We consider a set $X$ together with a family $N$ of filters in $X$ satisfying the following three conditions:

N1) to every $x\in X$ there corresponds uniquely a filter $\mathfrak{N}(x)$ each member of which contains $x$,

N2) a filter in $X$ containing an element of $N$ also belongs to $N$,

N3) for every $x\in X$, $\mathfrak{N}(x)\in N$.

We will denote such a space $X$ with $N$ by $(X;N)$ and call it a *space* simply.

A filter base $\mathfrak{f}$ in $X$ *converges* to $x$ in $X$ if and only if the filter generated by $\mathfrak{f}$ contains $\mathfrak{N}(x)$.[3]

A filter $\mathfrak{N}(x)$ and each of its members are called the *neighborhood system* of $x$ and a *neighborhood* of $x$ respectively.

A mapping $\varphi$ of a space $(X;N)$ into a space $(Y;M)$ is *continuous* if and only if for every $x\in X$ a filter generated by $\varphi(\mathfrak{N}(x))$ contains

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1) In this paper spaces are all $\mathcal{T}_1$ convergence spaces. See [4].

2) In that paper [7] the condition C6) is stated erroneously. It must be read as C6) of this paper and $\mathfrak{f}$ in the last two lines on page 464 must be a leg.

3) N1) with this definition of convergence is called $\mathcal{T}_1$ *convergence structure* of a space by Fisher.