147. *General Theory of Mappings*

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In his paper [1], J. R. Büchi considered the notion of functions on a set. Some of his results are true for the both set theories in the senses of G. Cantor and S. Leśniewski. In this paper, we concern with a theory of functions on a set in the sense of G. Cantor.

Let E, E' be two given sets, f a function from 2^E to $2^{E'}$, where 2^E, $2^{E'}$ denote the sets of all subsets of E, E' respectively.

J. R. Büchi [1] introduced a notion of *a pair of functions* $(f, \bar{f})$ as follows: f and $\bar{f}$ are a pair of functions, if, for any function f, there is a function $\bar{f}$ from $2^{E'}$ to 2^E such that $A' \cap f(A)=0$ implies $\bar{f}(A') \cap A=0$, where $A \in 2^E$, $A' \in 2^{E'}$. J. R. Büchi obtained some important properties on $(f, \bar{f})$ (see [1]). Among these properties, an important result is the representation of $\bar{f}: f(A')=\cap\{X \mid f(E-X) \subset E'-A'\}$.

If $(f, \bar{f})$ is a pair of functions, then for $\{A_\alpha\}$, $A_\alpha \subset E$, we have $f(\bigcup_\alpha A_\alpha)=\bigcup_\alpha f(A_\alpha)$ (see [1], p. 164). Hence f is a multiform mapping in the sense of Dubreil ([4]-[7]).

Further we have $\bar{f}(f(A)) \supset A$. To prove it, take an element x of A. Suppose that $\bar{f}(f(A)) \cap x=\phi$, then $f(A) \cap f(x)=0$, which contradicts to $f(x) \subset f(A)$.

For the empty set ϕ and E, we have $\bar{f}(f(\phi))=\phi$, $\bar{f}(f(E))=E$. Therefore the family $\mathfrak{M}$ of all subsets A of E such that $\bar{f}(f(A))=A$ is not empty.

Let $A=\bigcup_\alpha A_\alpha$, $A_\alpha \in \mathfrak{M}$, then we

$$\bar{f}(f(A))=\bar{f}(f(\bigcup_\alpha A_\alpha))=\bar{f}(\bigcup_\alpha f(A_\alpha))=\bigcup_\alpha \bar{f}(f(A_\alpha))=\bigcup A_\alpha=A.$$

Let $B=\bigcap_\alpha A_\alpha$, $A_\alpha \in \mathfrak{M}$, then

$$\bar{f}(f(B))=\bar{f}(f(\bigcap_\alpha A_\alpha)) \subset \bar{f}(\bigcap_\alpha f(A_\alpha)) \subset \bigcap_\alpha \bar{f}(f(A_\alpha))=\bigcap A_\alpha=B.$$

On the other hand, $B \subset \bar{f}(f(B))$ for any subset B of E.

For any subset $A \in \mathfrak{M}$, $(E-A) \cap \bar{f}(f(A))=(E-A) \cap A=\phi$. Hence

$$f(E-A) \cap f(A)=\phi.$$

This implies $\bar{f}(f(E-A)) \cap A=\phi$, and we have $\bar{f}(f(E-A)) \subset E-A$. Therefore, we have the following

Theorem 1. *The family $\mathfrak{M}$ of all subsets A such that $f(f(A))$*

1) In this Note, we shall assume that $f(x) \neq 0$ for every $x \in E$.