146. *Characterization of a De Morgan Lattice in Terms of Implication and Negation*

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The purpose of this Note is to give a characterization of De Morgan lattice in terms of implication and negation.

The notion of De Morgan lattice has been considered by Gr. C. Moisil [4] in the work mentioned in the reference included at the end of this Note and has been studied by J. A. Kalman [3] under the name of *distributive i-lattice*. A. Bialynicki-Birula and H. Rasiowas [2] have studied this type of lattice having the first element under the name of *quasi-boolean algebra*. The nomenclature used here is due to A. Monteiro [5].

A lattice can be defined as a system $(M, \cap, \cup)$ consisting of a non empty set M and two binary operations $\cup$, $\cap$ defined on M such that the following properties are verified:

L1. $x\cup y=y\cup x$, L′1. $x\cap y=y\cap x$,

L2. $x\cup(y\cup z)=(x\cup y)\cup z$, L′2. $x\cap(y\cap z)=(x\cap y)\cap z$,

L3. $x\cup(y\cap x)=x$, L′3. $x\cap(y\cup x)=x$.

A lattice is called a distributive lattice if it verifies the property:

D. $x\cup(y\cap z)=(x\cup z)\cap(x\cup z)$.

A distributive lattice is called a De Morgan lattice if a unary operation $-$ is defined on it such that the following two properties hold:

M1. $--x=x$,

M2. $-(x\cup y)=-x\cap -y$.

Theorem. *Let M be a non-empty set, $\rightarrow$ a binary operation and $-$ a unary operation defined on M such that the following properties are verified:*

A1. $x\rightarrow -y=y\rightarrow -x$,

A2. $(x\rightarrow -y)\rightarrow y=y$,

A3. $(x\rightarrow y)\rightarrow z=-((-x\rightarrow z)\rightarrow -(y\rightarrow z))$.

If we write $x\cup y=-x\rightarrow y$ and $x\cap y=-(x\rightarrow -y)$, then the system $(M, \cup, \cap, -)$ is a De Morgan lattice.

Proof. M1. $x=-(-x)$.

In order to prove this, let us see the following two relations:

a) $-x\rightarrow x=x$,

b) $x\rightarrow -x=-x$.