145. Almost Convergent Topology

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1. **Introduction.** The study in function space topologies mainly has been investigated in the space of continuous functions (see [3]). Recently Kolmogorov [2], Prokhorov [4], and Skorokhod [5] discussed topologies on the space of all discontinuous functions of the first kind in connection with a problem in probability theory. In the theory of probability, if the independent variable t is considered to be the time, then it is impossible to assume the existence of an instrument which will measure time exactly whence a comparatively weaker topology is considered (see [5]). In this paper for the above mentioned purpose almost convergent topology is considered and in the end of this paper one shows Skorokhod M-convergent is a special case of almost convergent topology.

(2.1) **Definition.** Let (X, L) and (Y, S) be topological spaces. For each pair of open sets $U \in L$ and $V \in S$, let

$$A(U, V) \equiv \{f \in Y^X : f(U) \cap V \neq \phi\}.$$

An almost convergent topology on Y^X is that topology which has as subbasis $\{A(U, V)\}$.

The following example provides a motivation to study the topology.

(2.2) **Example.** Let $f_n : [0, 1] \to R$ (reals with the usual topology)

$$f = \begin{cases} 0 & \text{on } [0, 1/2), \\ 1 & \text{on } [1/2, 1]. \end{cases}$$

$$f_n = \begin{cases} 0 \quad \text{on } [0, 2^n - 1/2^{n+1}), \\ 2^n x - (2^n - 1)/2 \quad \text{on } [(2^{n-1})/(2^{n+1}), (2^n + 1)/(2^{n+1})), \\ 1 \quad \text{on } [2^n + 1/2^{n+1}, 1] \end{cases}$$

for $n = 1, 2, \cdots$.

Since $f_n \notin P(1/2, S_\nu(1)) = \{f \in Y^X : f(1/2) \subset S_\nu(1)\}$, where $S_\nu(1)$ is the open sphere about 1 with the radius $\nu < 1/2$. $\{f_n\} \not\to f$ in the point open topology. However, $\{f_n\} \to f$ in the almost convergent topology (we denote as $\{f_n\} \xrightarrow{A} f$ from now on). As the relation with other topologies we have

(3.1) **Theorem.** *A-topology (Almost Convergent Topology) $\subset$ P-topology (point open topology).*