143. *On Almost Everywhere Convergence of Walsh-Fourier Series**)

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1. Introduction. L. Carleson [2] proved that the Fourier series of functions belonging to the class $L^2(-\pi, \pi)$ converge almost everywhere.

Combining the method of Carleson and the theory of interpolation of operators, R. A. Hunt [3] extended the result to the Fourier series of functions $f \in L^p(-\pi, \pi)$, $p>1$. In fact he proved three maximal theorems about partial sum of Fourier series. On the other hand P. Billard [1] applied the method of Carleson to Walsh-Fourier series of functions $f \in L^2(0, 1)$.

In the present paper, the author applies the Carleson-Hunt-Billard method to Walsh-Fourier series, and proves the analogues to Hunt's result.

Let $S_n(f)$ be the n-th partial sum of Walsh-Fourier series of integrable and periodic function $f(t)$ $(0\leq t\leq 1)$.

Let

$$Mf(t)=\operatorname{Sup}\{|S_n(f)| : n\geq 0\},$$

then the theorems of this paper are;

Theorem 1. *If* $1<p<\infty$, *then* $||Mf||_p\leq C_p||f||_p$.

Theorem 2. $||Mf||_1\leq C\int_0^1 |f(t)|(\log|f(t)|)^2dt+C$.

Theorem 3. *For any* $y>0$,

$$m\{t \in (0, 1)\,;\, Mf(t)>y\}\leq C\exp\{-Cy/||f||_\infty\}.$$

It is well known that these results imply the almost everywhere convergence of $S_n(f)$ to $f(\mathrm{t})$ for f in the respective function spaces.

2. Notation. Let $(r_1, r_2, \cdots, r_n, \cdots)$ and $(w_0, w_1, \cdots, w_n, \cdots)$ be the classical system of Rademacher and Walsh functions. For a positive integer n we define N_n by $2^{N_n}\leq n<2^{N_n-1}$ and write

(2.1) $n=\zeta_0+\zeta_1 2^1+\cdots+\zeta_{N_n}2^{N_n}(\zeta_j=0, 1\,;\, j=0, 1, 2, \cdots, N_n\,;\, \zeta_{N_n}=1).$

The Dirichlet kernel of Walsh system is defined by

(2.2) $$W_n(t)=w_0(t)+w_1(t)+\cdots+w_{n-1}(t).$$

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