142. Global Solution for an Initial Boundary Value Problem of a Quasilinear Hyperbolic System

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(Cnmm. by Kinjirô KUNUGI, M. J. A., Sept. 12, 1968)

§1. **Introduction.** We consider the following system of equations

(1) $\partial v/\partial t-\partial u/\partial x=0, \qquad \partial u/\partial t+\partial(a^2/v)/\partial x=0,$

which is the simplest equation in gas dynamics (Lagrangean form in the isothermal case: $p=a^2/v$, a is a constant>0), where v is the specific volume, u is the speed of the gas.

Here we consider the Cauchy problem in $t\geq 0$, $-\infty<x<+\infty$ for (1) with the initial values

(2) $v(0, x)=v_0(x),\ u(0, x)=u_0(x)$ for $-\infty<x<+\infty$

and also the piston problem (an initial boundary value problem) in $t\geq 0$, $x\geq 0$ for (1) with the boundary values

(3)
$$v(0, x)=v_0(x),\ u(0, x)=u_0(x) \quad \text{for } x\geq 0,$$
$$u(t, 0)=u_1(t) \quad \text{for } t\geq 0,$$

where $v_0(x)$, $u_0(x)$, $u_1(t)$ are bounded functions with locally bounded variation and $v_0(x)\geq\delta=\text{constant}>0$.

We see that the Cauchy problem (1), (2) and the piston problem (1), (3) have generalized solutions in the large. We use the Glimm's (or slightly modified) difference scheme [2] for the proof of the existence theorems.

There are many articles [1]-[8] which treat the existence theorem of the solution in the large for the initial value problem of the quasilinear hyperbolic system of equations, where the system is more general than in this paper, but the initial value is more restricted.

§2. **Cauchy problem.** Here we consider the Cauchy problem (1), (2). The definition of the generalized solution v, u of the Cauchy problem (1), (2) is the following: $v(t, x)$, $u(t, x)$ are bounded measurable functions and satisfy the integral identity

$$\iint_{t>0}(v\cdot f_t-u\cdot f_x)dtdx+\int_{t=0} v_0(x)f(0, x)dx=0$$
$$\iint_{t>0}(u\cdot g_t+(a^2/v)\cdot g_x)dtdx+\int_{t=0}u_0(x)g(0, x)dx=0$$

for any continuously differentiable functions f, g with compact support.

The system (1) is hyperbolic in $v>0$ and has the characteristics, the Riemann invariants and the nonlinearity as follows: