# 141. *The Characters of Some Induced Representations of Semisimple Lie Groups*

By Takeshi HIRAI
Department of Mathematics, Kyoto University

(Comm. by Kinjirô KUNUGI, M. J. A., Sept. 12, 1968)

**Introduction.** Let $G$ be a simply connected semisimple Lie group. Let $\mathfrak{g}_0$ be its Lie algebra and let $\mathfrak{g}_0=\mathfrak{k}_0+\mathfrak{p}_0$ be a Cartan decomposition of $\mathfrak{g}_0$, where $\mathfrak{k}_0$ is a maximal compact subalgebra of $\mathfrak{g}_0$. Let us fix arbitrarily a maximal abelian subalgebra $\mathfrak{h}_0^-$ of $\mathfrak{p}_0$. Let $\mathfrak{g}$ and $\mathfrak{h}^-$ be the complexifications of $\mathfrak{g}_0$ and $\mathfrak{h}_0^-$ respectively. Introduce a lexicographic order in the set of all roots of $\mathfrak{g}$ with respect to $\mathfrak{h}^-$ and let $\Delta$ be the set of all positive roots of $\mathfrak{g}$.

Fix an element $h_0\neq 0$ of $\mathfrak{h}_0^-$ and let $\Delta'$ be the set of all roots $\alpha\in\Delta$ zero at $h_0$ and $\Delta''$ the complement of $\Delta'$ in $\Delta$. Let $\mathfrak{h}_0'$ be the subalgebra of $\mathfrak{h}_0^-$ orthogonal to $\Delta'$. Consider the centralizer $S$ of $\mathfrak{h}_0'$ in $G$. Let $S_1$ be a subgroup of $S$ and let $s\to L_s$ $(s\in S_1)$ be a representation of $S_1$ by bounded operaters on a Hilbert space $E$. If $S_1$ and $L$ fulfill some conditions, we can construct canonically a representation of $G$ on a certain Hilbert space, starting from $L$ (see §1). After F. Bruhat [1] we call it induced representation of $L$ and denote it by $T^L$. He has studied in [1] a criterion of the irreducibility of $T^L$, when $L$ is of finite-dimensional. Our present purpose is (1) to obtain a sufficient condition on $S_1$ and $L$ for the existence of the characters of both $L$ and $T^L$, and (2) to express the character of $T^L$ by that of $L$ in the form of summation. This has been done in very special cases in [2], [3], and [4(b)].

**§1. Induced representations.** Let $\mathfrak{c}_0$ be the center of $\mathfrak{k}_0$ and put $\mathfrak{k}_0'=[\mathfrak{k}_0,\mathfrak{k}_0]$, then $\mathfrak{k}_0=\mathfrak{c}_0+\mathfrak{k}_0'$. For any $\alpha\in\Delta$, let $\mathfrak{g}_\alpha$ be the set of all elements $x$ of $\mathfrak{g}$ which fulfill

$$[h, x]=\alpha(h)x \quad (h\in\mathfrak{h}^-).$$

Put $\mathfrak{n}=\sum_{\alpha\in\Delta}\mathfrak{g}_\alpha$, $\mathfrak{n}'=\sum_{\alpha\in\Delta''}\mathfrak{g}_\alpha$, $\mathfrak{n}_0=\mathfrak{n}\cap\mathfrak{g}_0$, and $\mathfrak{n}_0'=\mathfrak{n}'\cap\mathfrak{g}_0$. Then $\mathfrak{n}_0$ and $\mathfrak{n}_0'$ are subalgebras of $\mathfrak{g}_0$. Let $K$, $H^-$, $D$, $K'$, $N$, and $N'$ be the analytic subgroups of $G$ corresponding to $\mathfrak{k}_0$, $\mathfrak{h}_0^-$, $\mathfrak{c}_0$, $\mathfrak{k}_0'$, $\mathfrak{n}_0$, and $\mathfrak{n}_0'$ respectively. Then $G=NH^-K$ is Iwasawa decomposition of $G$.

We assume that the subgroup $S_1$ fulfills that

$$S^0(D\cap Z)\subset S_1\subset S,$$

where $S^0$ is the connected component of the identity element of $S$ and $Z$ is the center of $G$. Moreover we assume on $L$ that $L_z$ is a scalar