140. *Linear Set of the Second Category with Zero Capacity*

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1. In Cluster Set Theory various kinds of scales are used to describe the smallness of sets of exceptional characters (cf. Noshiro [2]), among which main ones are:

(a) Set theoretical one (cardinal numbers);
(b) Topological one (Baire category);
(c) Measure theoretic one (linear measure, e.g.);
(d) Potential theoretic one (logarithmic capacity, e.g.).

As for the relationship among them, excluding known or trivial ones we are here interested in those between (b) and (c), and (b) and (d). Existence of linear sets of the first category with positive or zero linear measure (or logarithmic capacity) can be seen on taking suitable generalized Cantor linear sets. Existence of linear sets of the second category with positive linear measure (or logarithmic capacity) is trivial. Therefore the problem is whether there exists linear set of the second category with zero linear measure (or logarithmic capacity). We shall show in 3 and 4 that such a set of zero logarithmic capacity (and hence of zero linear measure) exists.

2. Before proceeding to the construction in 3 and 4 we pause here, for the sake of completeness and comparison, to state the existence proof due to Professor Kiyoshi Noshiro (orally communicated to the authors) of the linear set of the second category with zero linear measure.

Let Γ be the unit circle and U the unit disk. Take a sequence $\{z_n\}_1^\infty$ of points $z_n \in U$ with $\sum_1^\infty (1-|z_n|)<\infty$ such that the totality of accumulation points of $\{z_n\}_1^\infty$ coincides with Γ. Let f be the Blaschke product whose zero set is $\{z_n\}_1^\infty$. Then the cluster set $C_U(f, z_0)$ contains zero for all $z_0 \in \Gamma$. On the other hand the angular cluster set $C_\Delta(f, z_0)$ for every Stolz angle Δ at z_0 consists of only one point with modulus 1 for almost all z_0 in Γ. Therefore the set $J(f)=\{z_0 \in \Gamma \mid C_\Delta(f, z_0) = C_U(f, z_0)$ for every $\Delta\}$ is of linear measure zero. It is known that $J(f)$ is the complement of a set of the first category in Γ and hence it is of the second category (Collingwood [1]). The existence proof of sets of the second category with zero linear measure is herewith complete.