138. A Note on Nets and Metrization

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A collection $\mathcal{B}$ of subsets of a topological space X is a *net* for X if for each point x in X and open neighborhood U of x there exists a $B \in \mathcal{B}$ such that $x \in B \subset U$. A space with a σ-locally finite net is called a *σ-space* and a regular space with a countable net is called a *cosmic* space (A. Okuyama [15] and E. Michael [7]). We assume at least T_1 for every topological space throughout this paper. For terminology not defined here, see J. Nagata [11].

A collection $\mathcal{F}$ of closed subsets of a topological space X is a *ct-net* for X if for any different points x, x' of X there is an $F \in \mathcal{F}$ such that $x \in F$ and $x' \notin F$. A space with a σ-closure preserving *ct-net* is called a *$\sigma^{\#}$-space*.

A base $\mathcal{B}$ for a space X is *point countable* if each point x of X is in at most countably many members of $\mathcal{B}$.

A space is *semi-metrizable* if there is a distance function d for X such that (i) for each x and y in X, $d(x, y)=d(y, x)\geqslant 0$ and $d(y, y)=0$ only if $x=y$, (ii) for $x \in X$ and $M \subset X$, $\inf\{d(x, y) \mid y \in M\}=0$ iff $x \in \bar{M}$.

A space X has a *G_δ-diagonal* if the diagonal in $X \times X$ is a G_δ-set.

Let $\{\mathcal{U}_i \mid i=1, 2, \cdots\}$ be a sequence of covers of a space X satisfying the condition:

(M) If $\{K_i \mid i=1, 2, \cdots\}$ is a decreasing sequence of non-empty sets of X such that $K_i \in St(x_0, \mathcal{U}_i)$ for each i and for some fixed point x_0 of X, then $\bigcap_{i=1}^{\infty} \bar{K}_i \neq \phi$.

A space is a *$w\Delta$-space* if there exists a sequence $\{\mathcal{U}_i\}$ of open covers satisfying (M) (C. Borges [3]). A space is an *M-space* if there exists a normal sequence $\{\mathcal{U}_i\}$ of open covers satisfying (M). (K. Morita [8]). A space is an *M^*-space* if there exists a sequence $\{\mathcal{U}_i\}$ of locally finite closed covers satisfying (M) (T. Ishii [6]). A space is an *$M^{\#}$-space* if there exists a sequence $\{\mathcal{U}_i\}$ of closure-preserving closed covers satisfying (M).

The results of this paper have been partially announced in [17] and [18].

A. Okuyama [16] has shown that a collectionwise normal σ-space has a σ-discrete net. However we have the following

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