# 137. *Characteristic Classes for Spherical Fiber Spaces*[1)]

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**1. Statement of results.** Let $SF=SG=\lim_{n} SG(n)$, $SG(n)=\{f:S^n \to \text{degree } 1\}$, $B_{SF}$ be the classifying space of $SF$. Our purpose is to determine $H_*(B_{SF}, Z_p)$ as a Hopf-algebra over $Z_p$, where $p$ is an odd prime number. Coefficient is always $Z_p$, and we omit it in the sequel. Let $Q_0(S^0)=\lim_{\overrightarrow{n}} \Omega_0^n S^n$. Then $Q_0(S^0)$ has the same homotopy type of $SF$. Let $i:Q_0(S^0)\to SF$ be the homotopy equivalence. Dyer-Lashof determined $H_*(Q_0(S^0))$ as an algebra over $Z_p$. $H_*(Q_0(S^0))$ is a free commutative algebra generated by $x_J, J\in H$, where $H=\{J=(\varepsilon_1, j_1, \cdots, \varepsilon_r, j_r)\}$ satisfies the following properties: 1) $r\geq 1$, 2) $j_i\equiv 0, (p-1)$, 3) $j_r\equiv 0$ $(2(p-1))$, 4) $(p-1)\leq j_1\leq j_2 \cdots \leq j_r$ 5), $\varepsilon_i=0$ or 1, 6) if $\varepsilon_{i+1}=0$ then $j_i/(p-1)$ and $j_{i+1}/(p-1)$ are even parity, if $\varepsilon_{i+1}=1$ then $j_i/(p-1)$ and $j_{i+1}/(p-1)$ are odd parity. There is a continuous map $h_0:L_p\to Q_0(S^0)$, and $x_j\equiv h_{0*}(e_{2j(p-1)})$, where $e_i\in H_i(L_p)$ is a generator, and $x_I \equiv x_{(\varepsilon_1, j_1, \cdots, \varepsilon_r, j_r)}\equiv \beta_p^{\varepsilon_1}Q_{j_1}\cdots \beta_p^{\varepsilon_{r-1}}Q_{j_{r-1}}\beta_p^{\varepsilon_r}x_{j_r}$, where $Q_j$ is the extended power operation defined by Dyer-Lashof. We identify $H_*(Q_0(S^0))$ and $H_*(SF)$ by $i_*$ as a $Z_p$-module and we denote $\tilde{x}=i_*(x)$, if $x\in H_*(Q_0(S^0))$.

**Theorem 1.** *$H_*(SF)$ is a free commutative algebra generated by $\tilde{x}_J$; $J\in H$. Even though $i_*$ is not a ring homomorphism.*

Let $H_1$ be the subset of $H$ consisting of $J=(\varepsilon_1, j_1, \cdots, \varepsilon_r, j_r)$, such that $j_1\neq p-1$, and $r\geq 2$. Let $H_2=\{(\varepsilon, p-1, 1, j)\}\subseteq H$. And let $H_i^+ =\{J\in H_i, \deg(x_J)=\text{even}\}$, $H_i^-=\{J\in H_i, \deg(x_J)=\text{odd}\}$ $i=1,2,\cdots$. Let $j;B_{SO}\to B_{SF}$ be the inclusion map. Then by Peterson-Toda, $H_*(B_{SO})/\ker j^*\cong Z_p[z_1, z_2, \cdots]$, where $\deg(z_j)=2j(p-1)$, and $\Delta(z_j)=\sum_{j_1+j_2=j} z_{j_1}\otimes z_{j_2}$, $z_0=1$. Let $\tilde{z}_j=j_*(Z_j)\in H(B_{SF})$.

**Theorem 2.** *$H_*(B_{SF})=Z_p[\tilde{z}_1, \tilde{z}_2, \cdots]\otimes\Lambda(\sigma\tilde{x}_1, \sigma\tilde{x}_2, \cdots)\otimes C_*$. $C_*$ is a free commutative algebra generated by $\tilde{x}_J$, $J\in H_1\cup H_2$. $\sigma; H_*(SF)\to H_*(B_{SF})$ is suspension. $\sigma\tilde{x}_j, \sigma\tilde{x}_J$ are primitive elements, and $\Delta(\tilde{z}_j) = \sum_{j_1+j_2=j}\tilde{z}_{j_1}\otimes\tilde{z}_{j_2}$.*

*$H^*(B_{SF})=Z_p[q_1, q_2, \cdots]\otimes\Lambda(\Delta q_1, \Delta q_2, \cdots)\otimes C$. $C=\bigotimes_{I\in H_1^+\cup H_2^+}\Lambda((\sigma\tilde{x}_I)^*)$ $\bigotimes_{J\in H_1^-\cup H_2^+}\Gamma_p[(\sigma\tilde{x}_J)^*]$, where $(\ )^*$ denotes dual elements, where $q_j$ is the $j$-th Wu-class, $j=1,2,\cdots$.*

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