133. *Criteria for Oscillation of Solutions of Differential Equations of Arbitrary Order*[1)]

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(Comm. by Zyoiti SUETUNA, M. J. A., Sept. 12, 1968)

H. Onose extending a result of the author [2], gave in [7] a sufficient condition for all solutions of the equation

$$(*) \qquad x^{(n)}+p(t)g(x, x', \cdots, x^{(n-1)})=0$$

to oscillate, provided that n is even and g homogeneous of degree $2s+1$.

Here we improve Onose's result considerably, by assuming quite weaker conditions which guarantee the oscillation of all solutions of $(*)$, and moreover, we consider the case $n=$odd. Thus, we also improve a result due to Howard ([1], Theorem 2), and generalize results of Ličko and Švec [5], and Mikusiński [6].

All functions considered are supposed to be continuous on their domains, and such that they guarantee the existence of solutions of $(*)$ for all large t (n will always be supposed to be >1). In what follows, we consider only such solutions which are nontrivial for all large t. By an oscillatory solution of $(*)$, we mean a solution with arbitrarily large zeros.

1. The following theorem has been proved in [4]:

Theorem 1. *For n even, let $(*)$ satisfy the following assumptions:*

(i) $p: I\rightarrow \boldsymbol{R}_+=(0, +\infty), \quad I=[t_0, +\infty), \quad t_0\geq 0,$ *and*

(S) $$\int_{t_0}^{\infty} t^{n-1}p(t)dt=+\infty;$$

(ii) $g: \boldsymbol{R}^n\rightarrow \boldsymbol{R}=(-\infty, +\infty), \quad x_1 g(x_1, x_2, \cdots, x_n)>0$ *for every* $(x_1, \cdots, x_n)\in \boldsymbol{R}^n$ *with* $x_1\neq 0$;

then every bounded solution of $(*)$ is oscillatory.

Now we show that an analogous result holds for the case $n=$odd. In fact, we establish the following

Lemma. *Suppose that n is odd, and that the functions p, g satisfy the hypotheses of Theorem 1; then every bounded solution of $(*)$ is oscillatory, or tends to zero monotonically as $t\rightarrow +\infty$.*

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1) This research was partially supported by a NATO grant.