132. Real-valued Measurable Cardinals and Σ_1^1-Transcendency of Cardinals*)

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(Comm. by Zyoiti SUETUNA, M. J. A., Sept. 12, 1968)

In this paper, we shall prove $\sum_1^1$-transcendency of cardinals[1) under the assumption of existence of real-valued measurable cardinal,[2) applying the results and difinitions used in [2] and [3].

Let I be an ideal over a set A. The equivalence relation between two subsets B and C of A is defined by

$$B \sim C \equiv B - C - B \in I \wedge B - C \in I.$$

By $[B]$ we donote the equivalence class including B. And $[A]$ and $[\phi]$ are sometimes abbreviated as $\mathbf{1}$ and $\mathbf{0}$ respectively. The relation $[B] > [C]$ is defined by $[B] > [C] \equiv BC \notin I \wedge C - B \in I$.

An ideal I is called a-complete if

$$[A_\nu] = \mathbf{0} \text{ for all } \nu < a \text{ implies } [\bigcup_{\nu < a} A_\nu] = \mathbf{0}.$$

The character of I is defined to be the smallest ordinal a such that I is not a-complete, and it is denoted by $\mathrm{ch}(I)$.

An ideal I is called a-saturated if

$$[A_\nu] > \mathbf{0},\ [A_\nu \cap A_\mu] = \mathbf{0} \text{ for all } \nu \neq \mu, \text{ and } \nu,\ \mu < b \text{ imply } b < a.$$

The saturation number of I is defined to be the smallest ordinal a such that I is a-saturated, and it is denoted by $\mathrm{sat}(I)$.

Let I be an ideal over $\aleph_\tau$. And let $\mathfrak{A}$ be a set of functions in $\mathrm{On}^{\aleph_\tau}$ (On is the class of all ordinal numbers). A function f is said to be incompressible (cf. [3]) with respect to $\mathfrak{A}$ if the following conditions are satisfied:

(1) $[\{\nu : g(\nu) < f(\nu)\}] = \mathbf{1}$ for every $g \in \mathfrak{A}$,

(2) if $[\{\nu : h(\nu) < f(\nu)\}] > \mathbf{0}$, then, $[\{\nu : h(\nu) \leq g(\nu)\}] > \mathbf{0}$ for some $g \in \mathfrak{A}$.

The following lemma is proved easily. (cf. [3]).

Lemma 1. *Let I be an ideal over $\aleph_\tau$ such that* $\mathrm{sat}(I) \leq \mathrm{ch}(I)$ ($\aleph_0 < \mathrm{ch}(I)$). *And let $\mathfrak{A}$ be a set of functions in* $\mathrm{On}^{\aleph_\tau}$. *Then there is an incompressible function with respect to $\mathfrak{A}$.*

Now we shall define a function $a^* \in On^{\aleph_\tau}$ by the induction on a as one of incompressible functions with respect to $\{b^* : b < a\}$. And $a^*(\nu)$

*) This work is partially supported by Matsunaga Science Foundation.

1) Cf. [2], [5].

2) Cf. [3], [6].