131. A Note on Semi-prime Modules. II

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The main purpose of this note is to prove the following two theorems:[1]

Theorem A. *Let R be a semi-prime Goldie ring, let Q be the right quotient ring of R, and let R_i $(i=1, \cdots, t)$ be the minimal annihilator ideals*[2] *of R. Let M be a semi-prime R-module, let M_i be the subisomorphism classes of basic submodules*[3] *of M which corresponds to R_i and let J_i be a uniform right ideal contained in R_i $(i=1, \cdots, t)$. Then*

(i) *There exists an element $x_i \in M_i$ such that $I_i = \mathrm{Hom}_R(x_iJ_i, x_iJ_i)$ is a right Ore domain. The ring D_i $\mathrm{Hom}_R(x_iJ_iQ, x_iJ_iQ)$ is the right quotient division ring of I_i $(i=1, \cdots, t)$.*

(ii) *The ring $I=\mathrm{Hom}_R(N, N)$ is isomorphic onto $I_1 \oplus \cdots \oplus I_t$, where $N = x_1J_1 \oplus \cdots \oplus x_tJ_t$.*

(iii) *The ring $D=\mathrm{Hom}_R(NQ, NQ)$ is the right quotient ring of I and is isomorphic onto $D_1 \oplus \cdots \oplus D_t$.*

Theorem B. *Let R be a Goldie ring. If M is a semi-prime R-module, then M contains N, which is a direct sum of uniform submodules and R is contained in a semi-prime ring B such that the pair (B, N) has the double centralizer property. The submodule N may be chosen to be of the form $x_1J_1 \oplus \cdots \oplus x_tJ_t$, where $x_i \in M_i$ and J_i is a uniform right ideal in R_i $(i=1, \cdots, t)$.*

1. Proof of Theorem A. Lemma 1. *Let M be a semi-prime R-module and let Q be the right quotient ring of R. Then the injective envelope $\tilde{M}$ of M is MQ.*

Proof. Let $x = mc^{-1}$ be a non-zero element of MQ. Then $xc = m \in M \cap xR$, which implies that MQ is an essential extension of M. Suppose that M' is an essential extension of M, then $M'^{\blacktriangle}=0$ and M' is faithful. Hence, by Proposition 1 in [7], M' is also semi-prime. By Proposition 4.1 in [3], we have $MQ = M'Q \supseteq M'$, which proves the lemma.

Since MQ is the injective envelope of M and $M^{\blacktriangle}=0$, we may

1) Throughout this paper, definitions and notations are used in the same sense as in [7]. R will denote a right Goldie ring and all R-modules will mean faithful right R-modules.

2) Cf. [5. p. 215].

3) Cf. [7. Theorem 7].